Soluciones a algunos problemas gráficos correspondientes a los problemas de Matemáticas II para 2^{o} de Bachillerato

Pedro González Ruiz

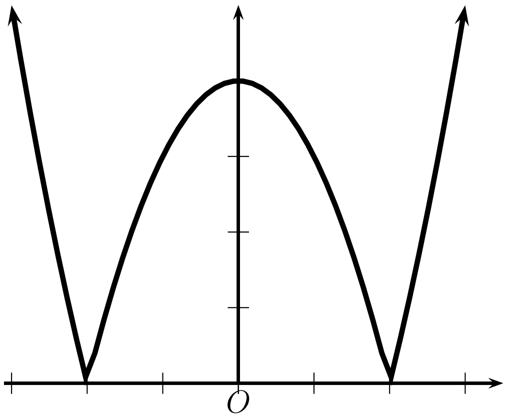
Catedrático de Matemáticas

Versión 13: septiembre de 2018

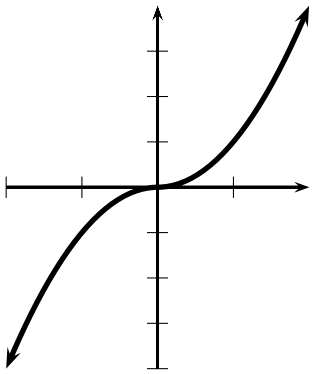
Capítulo 2

Sección 2.2

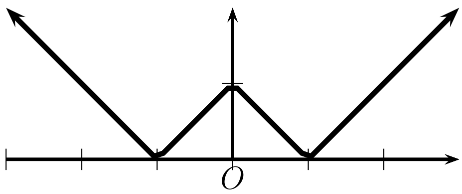
6.



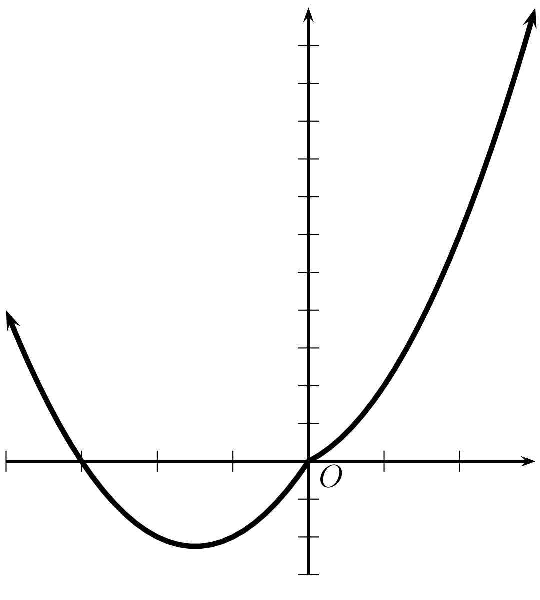
7.



8.

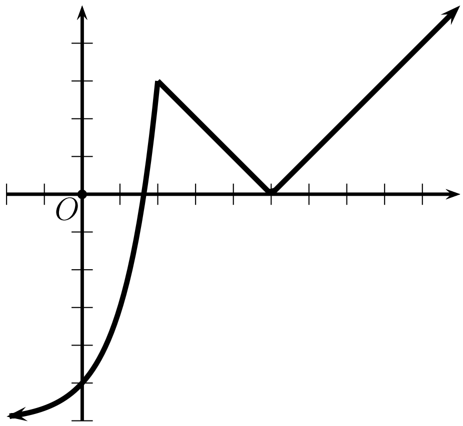


9.

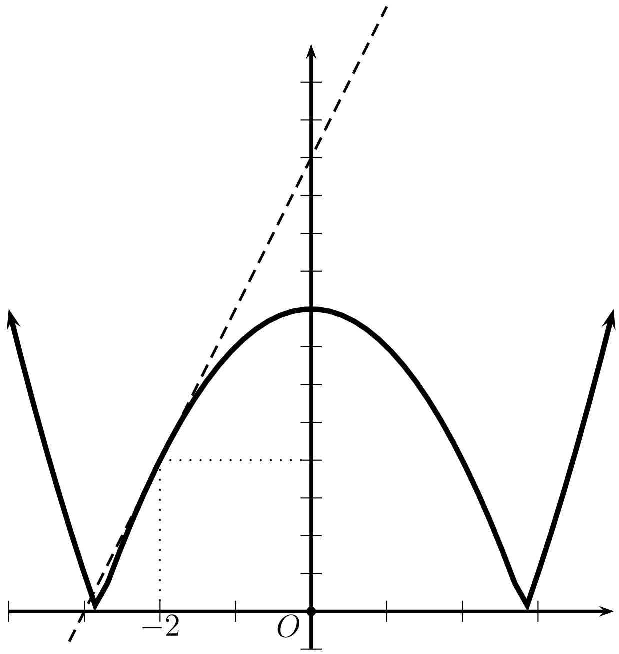


Sección 2.8

7.



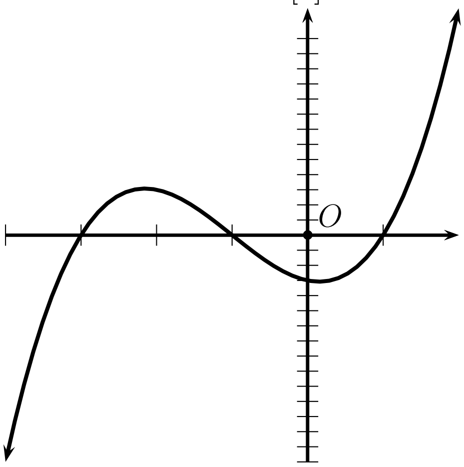
10.



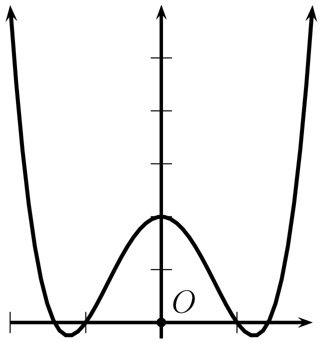
Capítulo 4

Sección 4.1

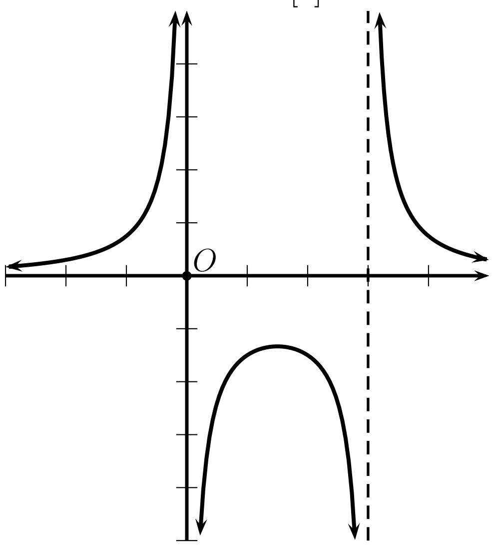
[1]



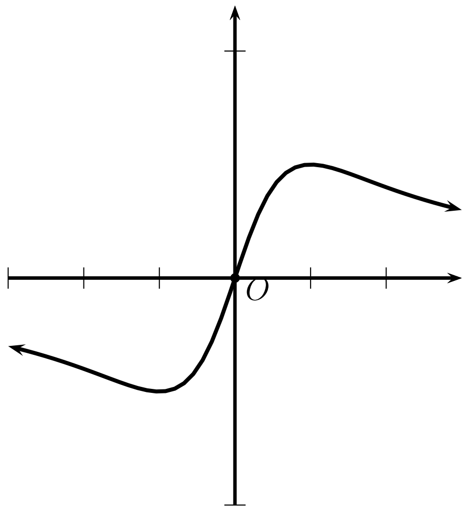
[2]



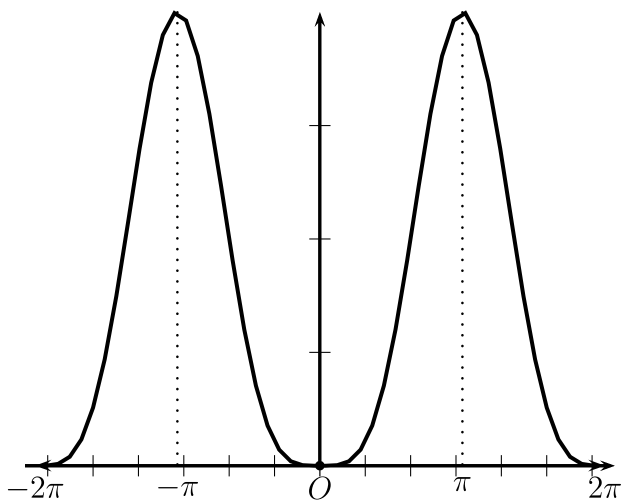
[3]



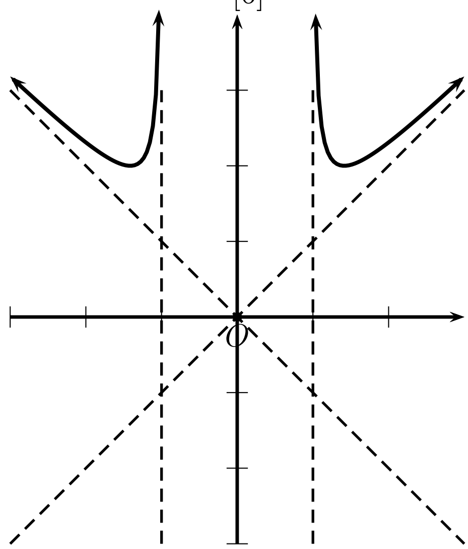
[4]



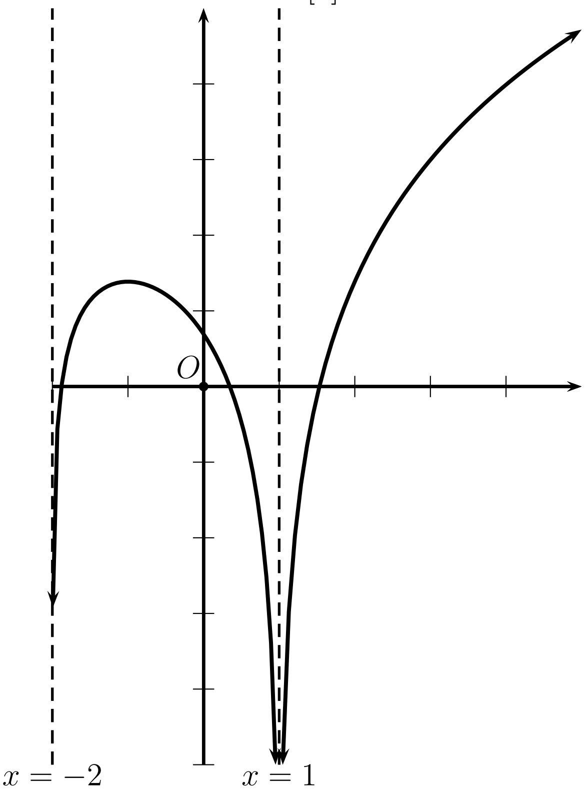
[5]



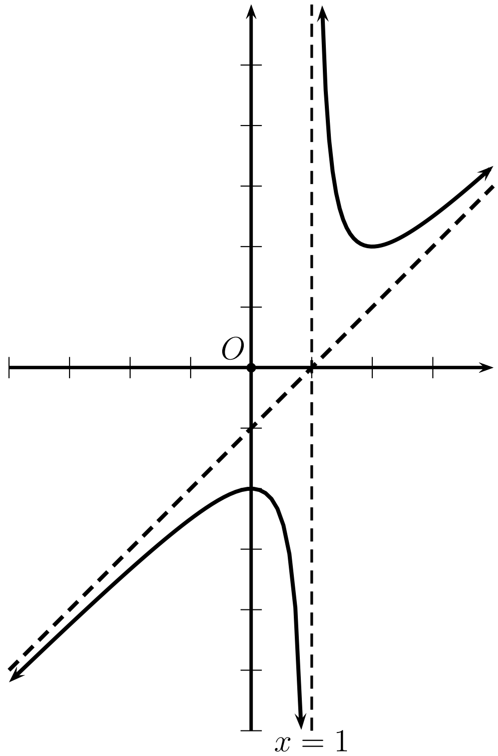
[6]



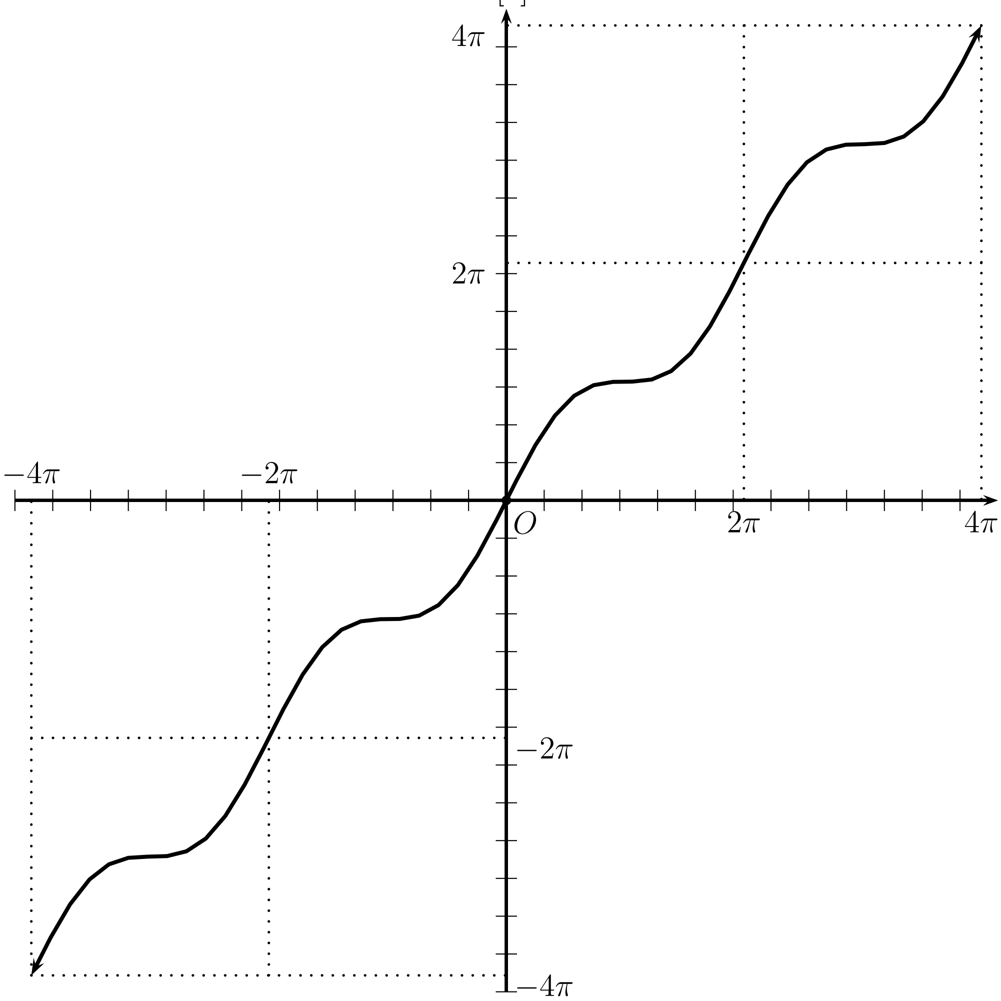
[7]



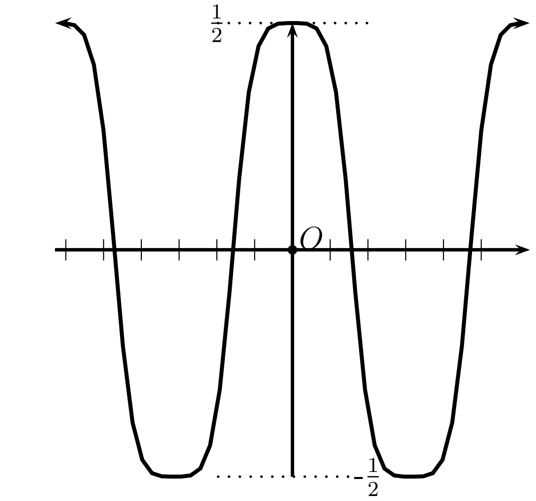
[8]



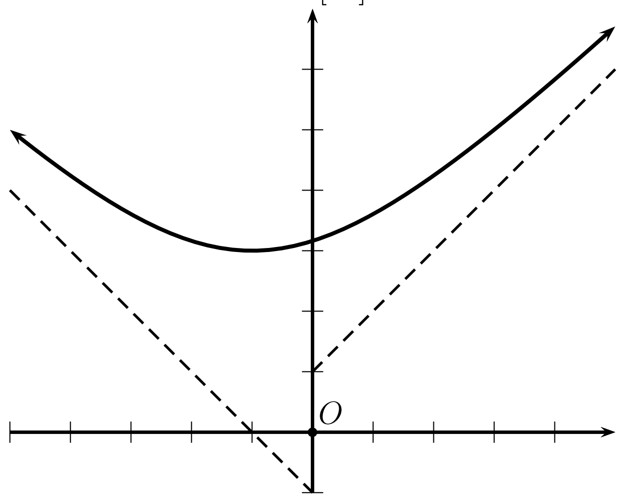
[9]



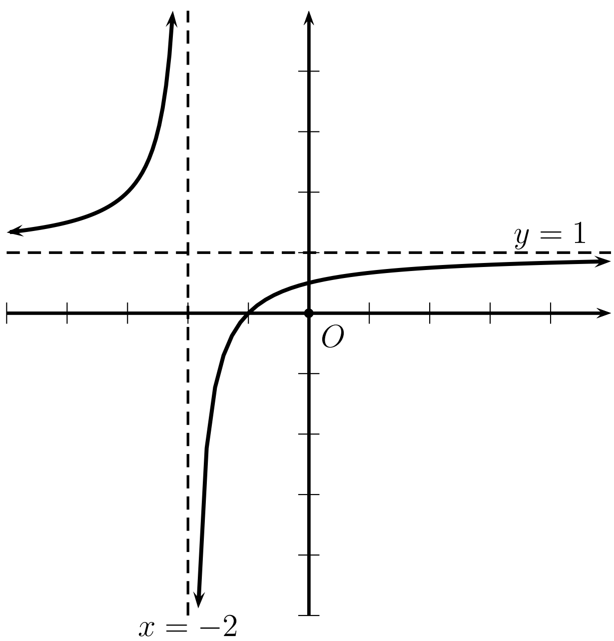
[10]



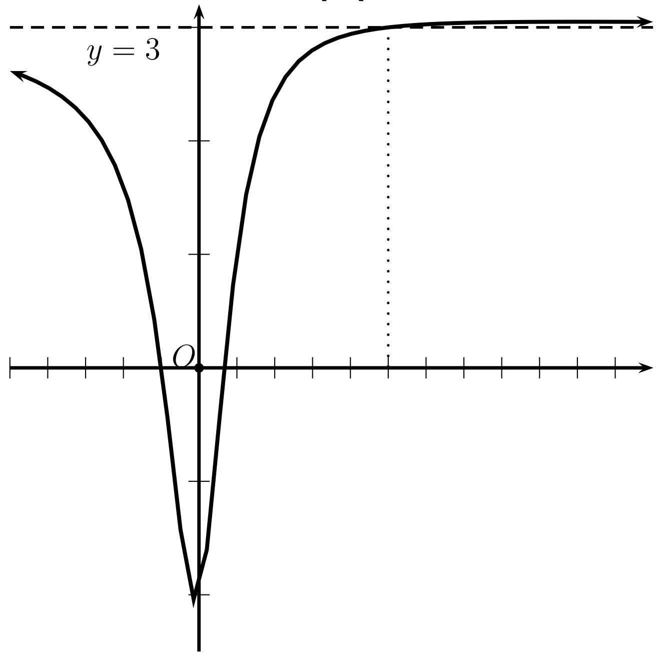
[11]



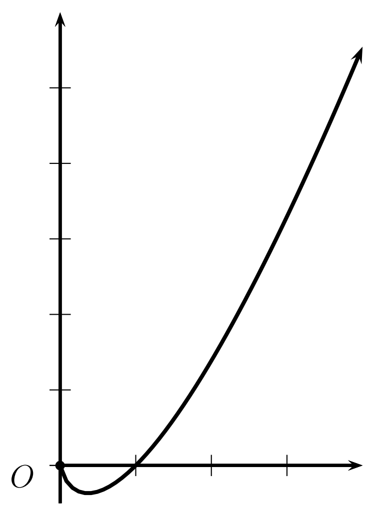
[12]



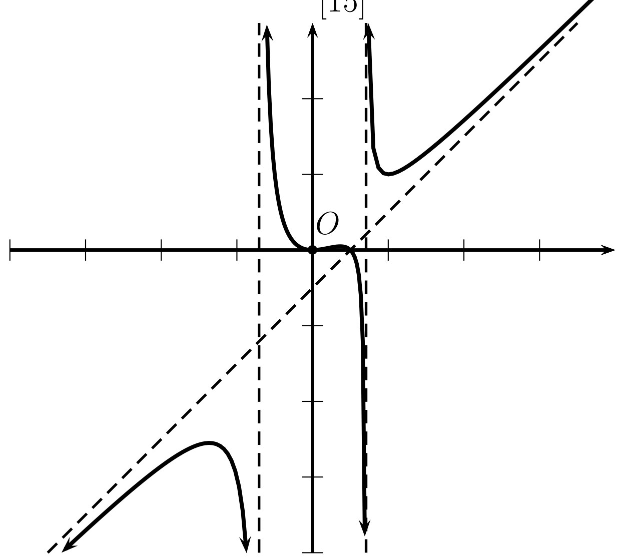
[13]



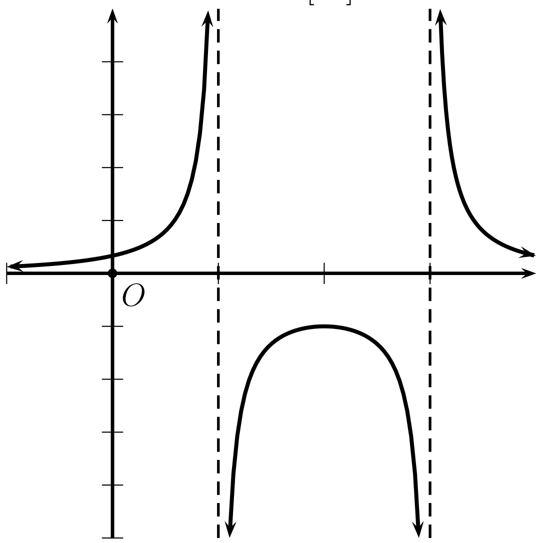
[14]



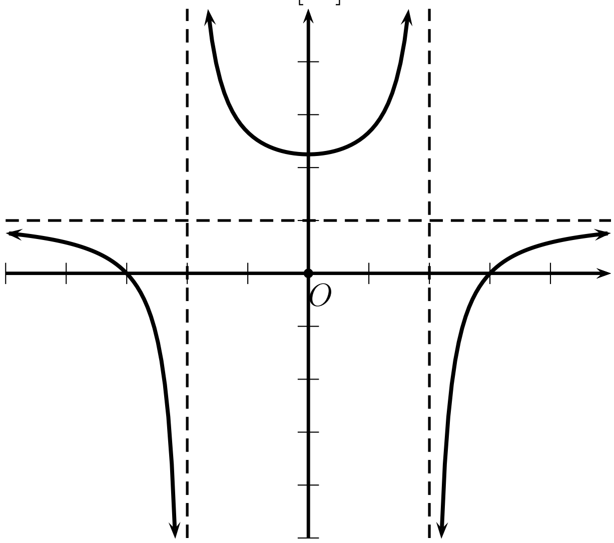
[15]



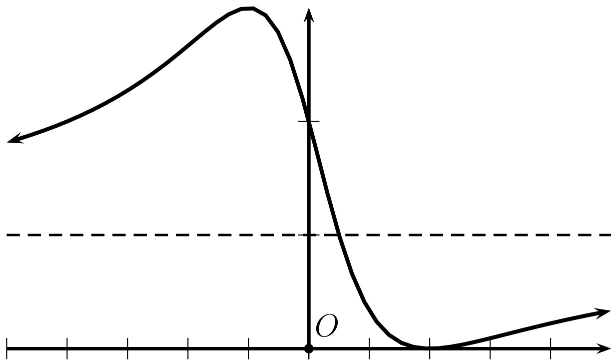
[16]



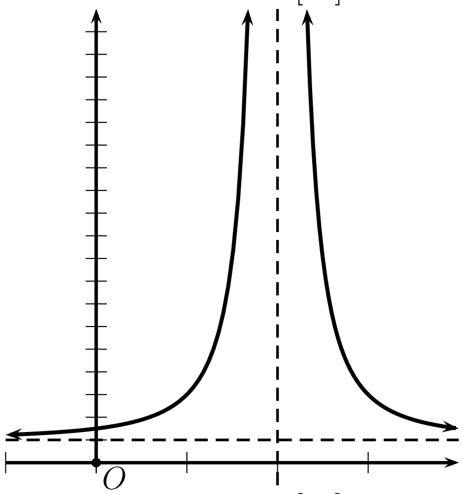
[17]



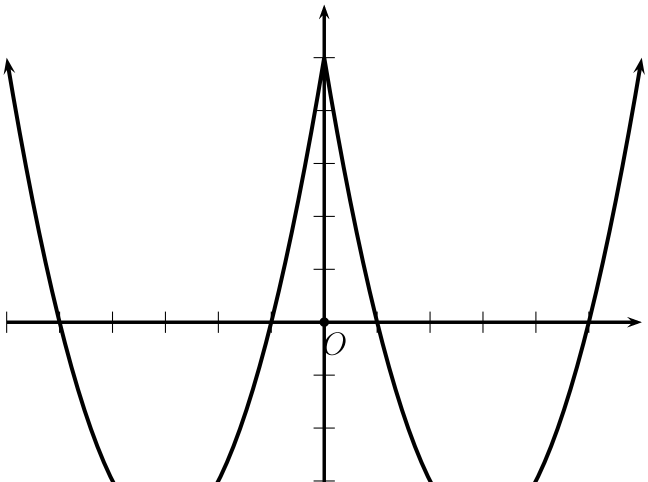
[18]



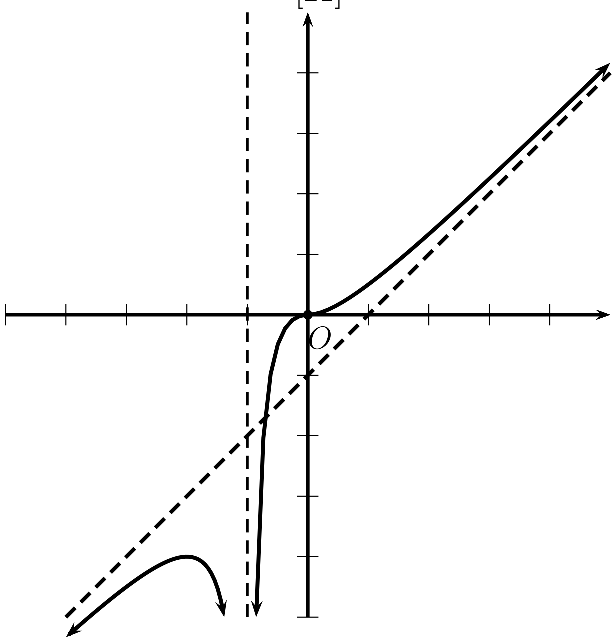
[19]



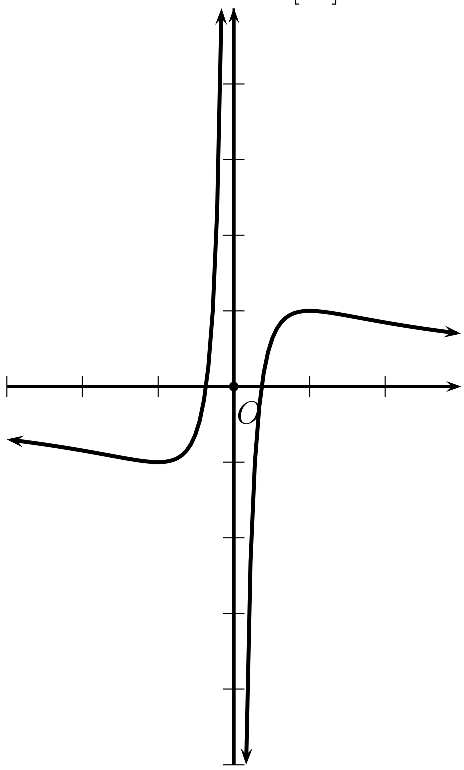
[20]



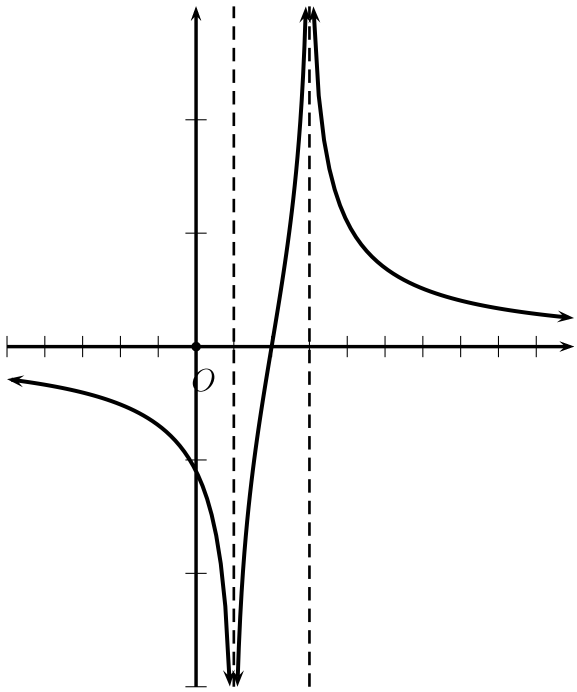
[21]



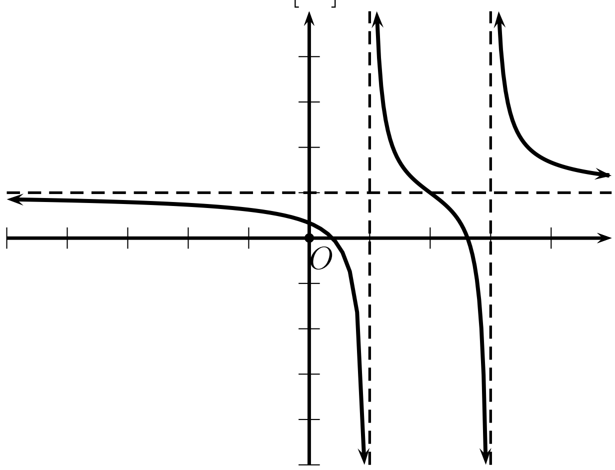
[22]



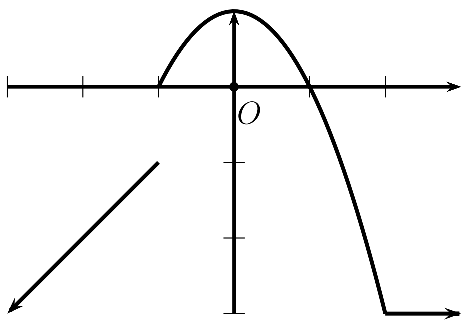
[23]



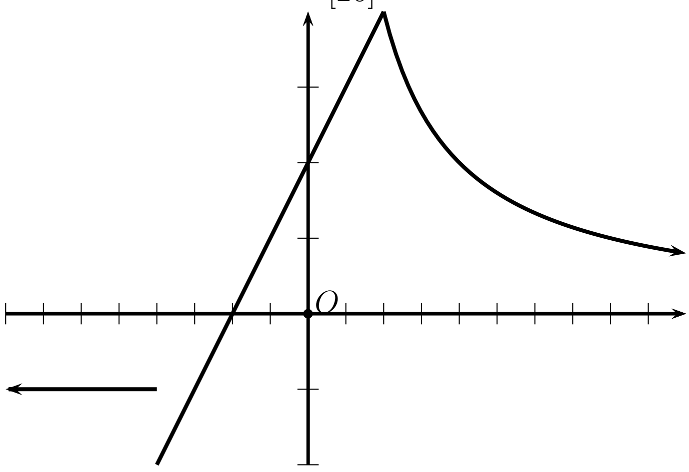
[24]



[25]

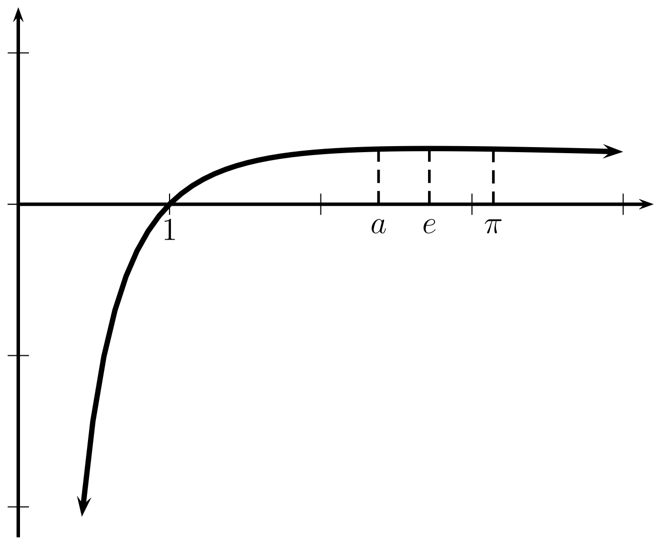


[26]

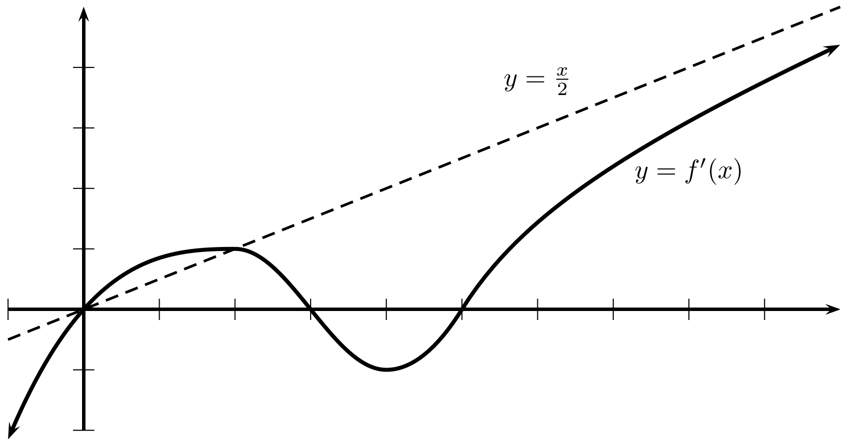


Sección 4.2

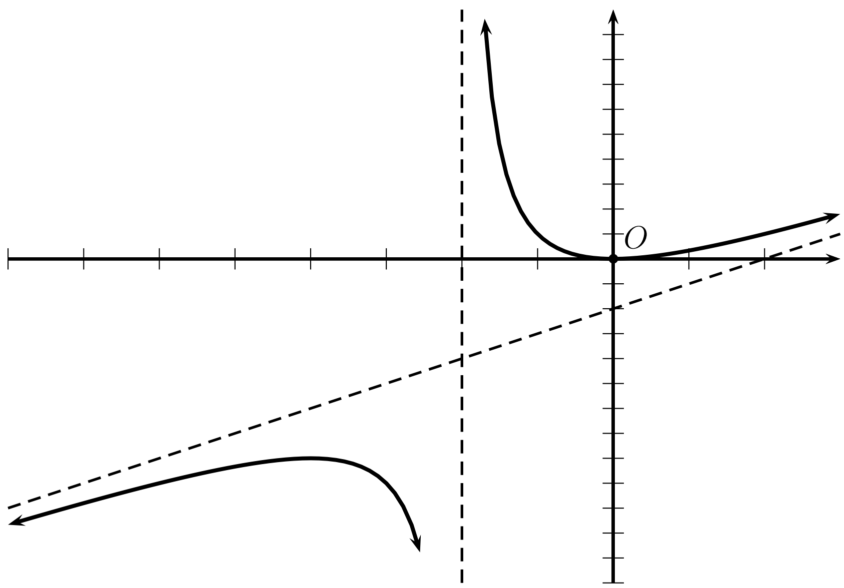
1.



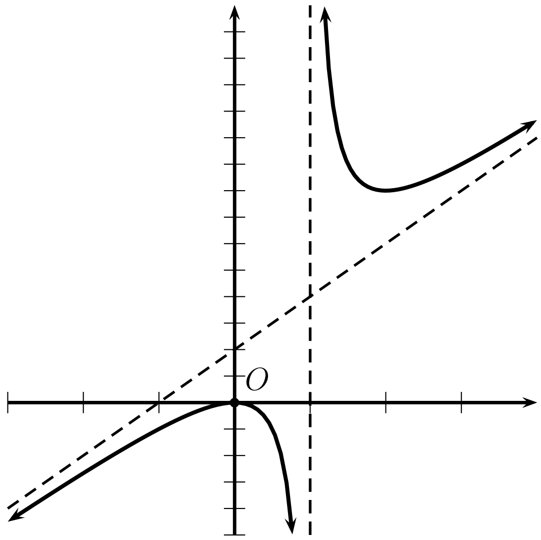
2.



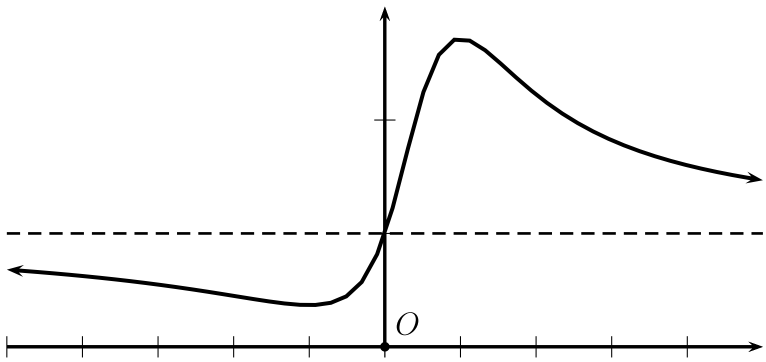
4.



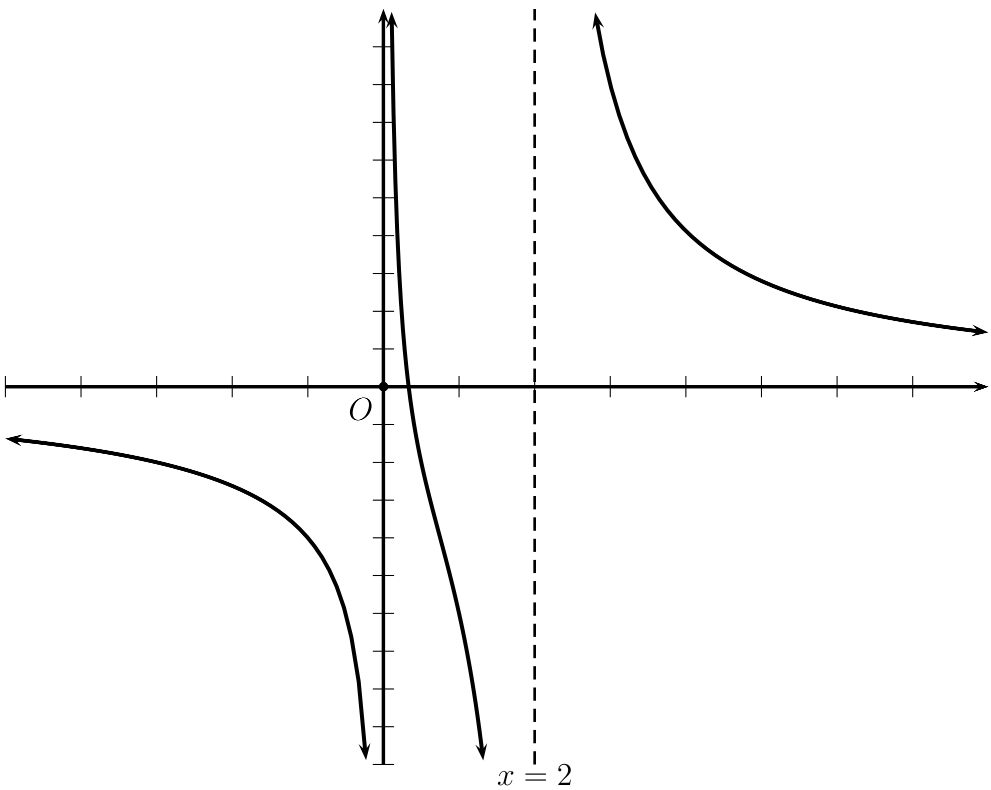
5.



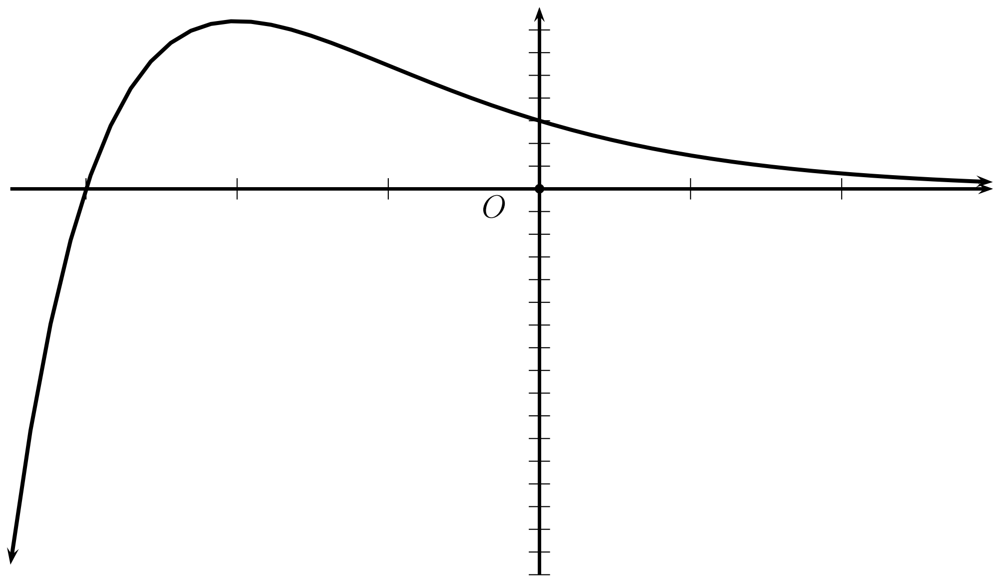
6.



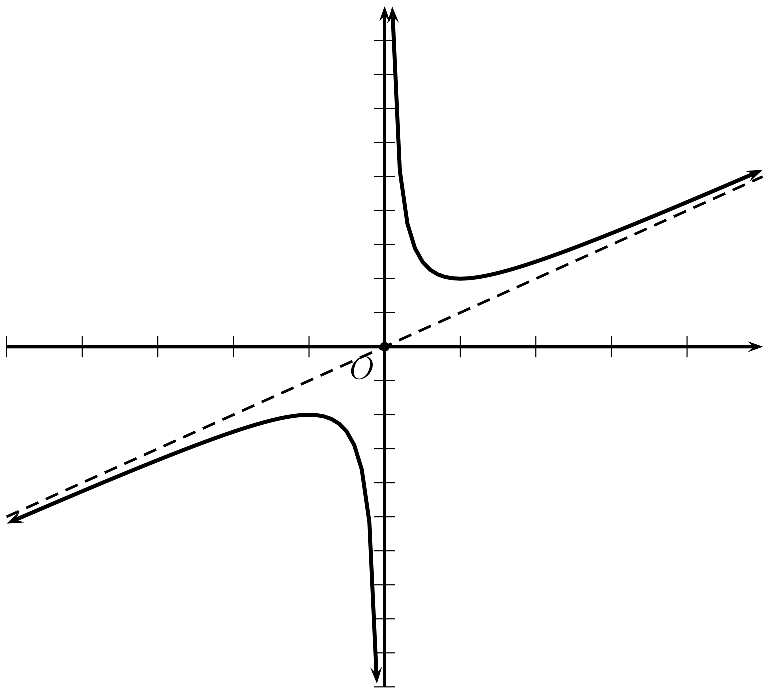
7.



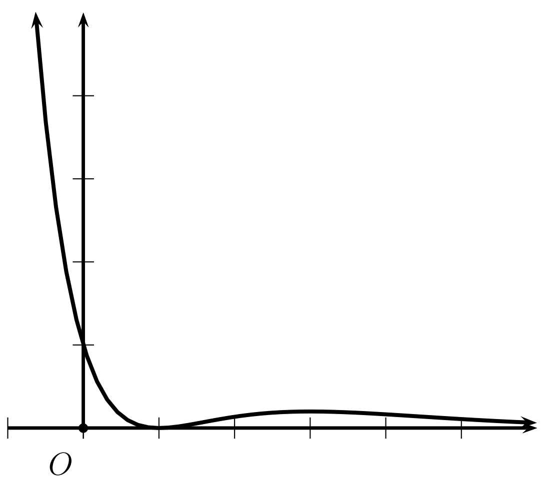
8.



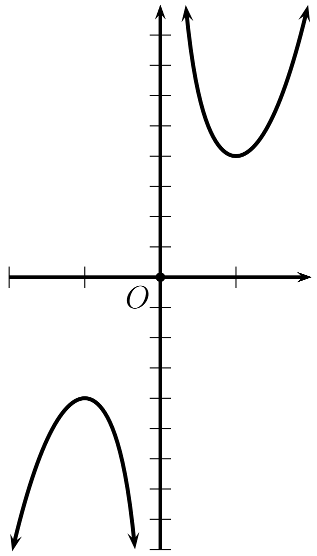
9.



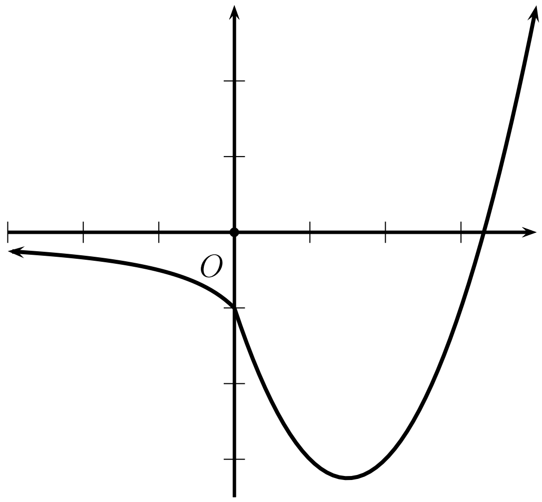
10.



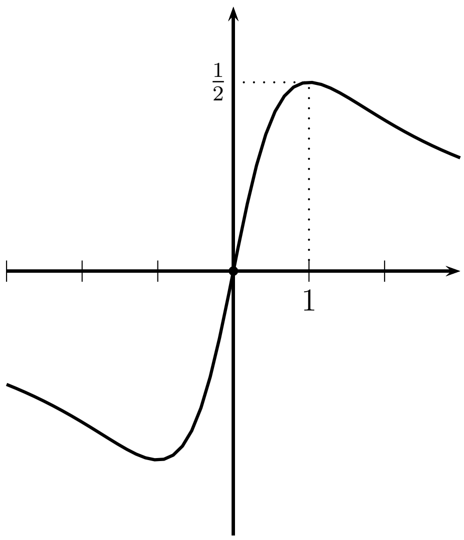
11.



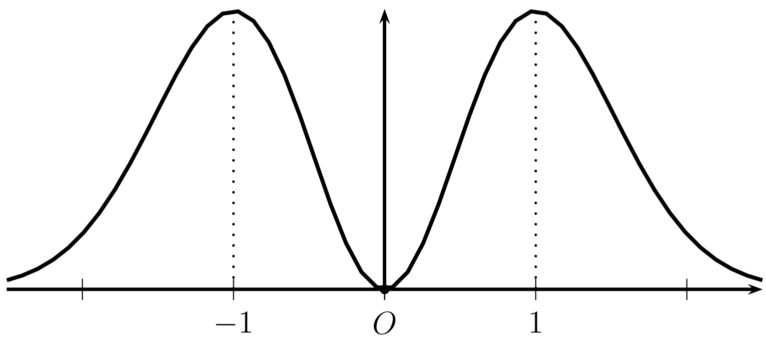
14.



20.

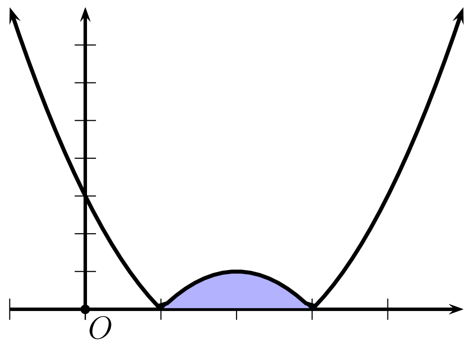


21.



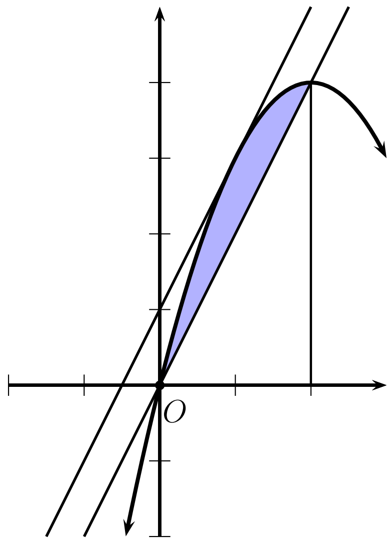
Capítulo 6

3.



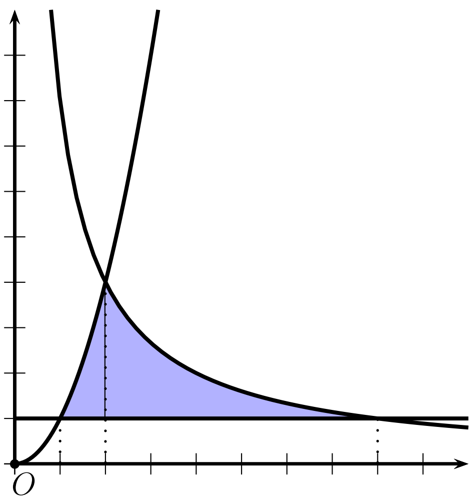
$$S = \int_1^3 |x^2 - 4x + 3|\, dx =$$
$$= \int_1^3 -(x^2 - 4x + 3)\, dx = \frac{4}{3}$$

4.



$$S = \int_0^2 \left[(-x^2+4x) - 2x\right] dx = \frac{4}{3}$$

5.



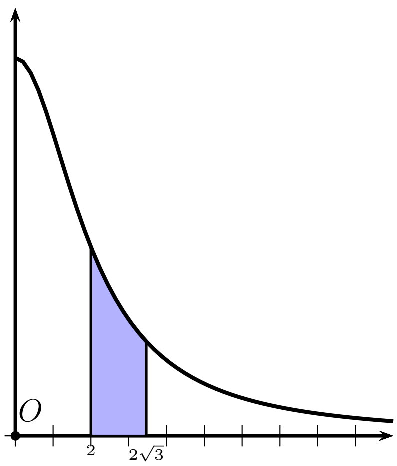
Respecto al eje X:

$$S = \int_1^2 (x^2-1)\,dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx =$$
$$= \frac{4}{3} + (16\ln 2 - 6) = 16\ln 2 - \frac{14}{3}$$

Más fácil con respecto al eje Y:

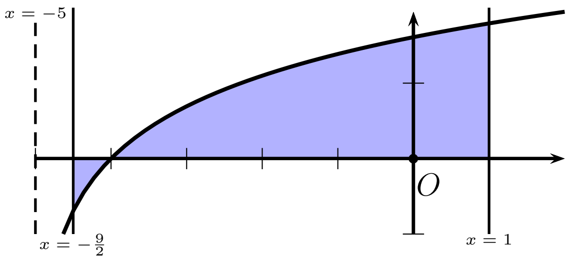
$$S = \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dy = 16\ln 2 - \frac{14}{3}$$

7.



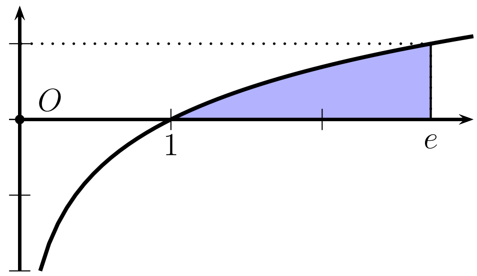
$$S = \int_2^{2\sqrt{3}} \frac{1}{4+x^2}\,dx = \frac{\pi}{24}$$

8.



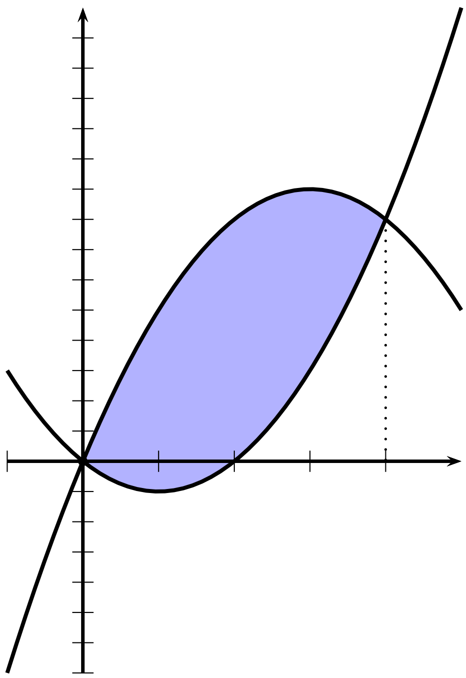
$$S=\int_{-\frac{9}{2}}^{-4}-\ln(x+5)\,dx+\int_{-4}^{1}\ln(x+5)\,dx=$$
$$=\left(-\frac{1}{2}\ln 2+\frac{1}{2}\right)+(6\ln 6-5)=$$
$$=6\ln 6-\frac{1}{2}\ln 2-\frac{9}{2}$$

9.



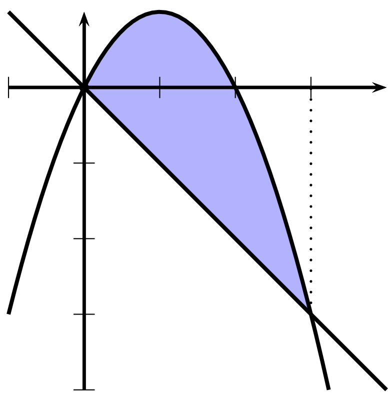
$$S=\int_{1}^{e}\ln x\,dx=1$$

10.



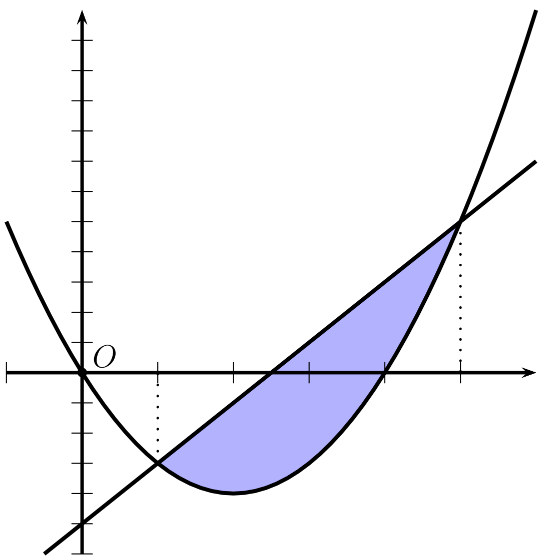
$$S=\int_{0}^{4}\left[(6x-x^2)-(x^2-2x)\right]dx=\frac{64}{3}$$

12.



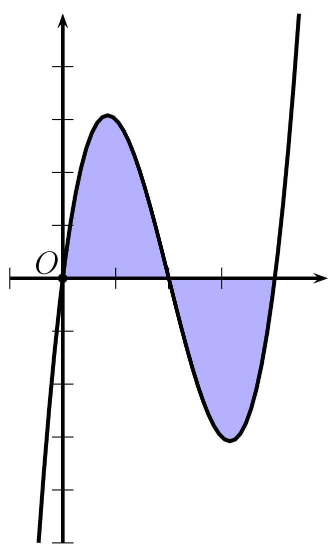
$$S = \int_0^3 \left[(2x - x^2) + x\right] dx = \frac{9}{2}$$

14.



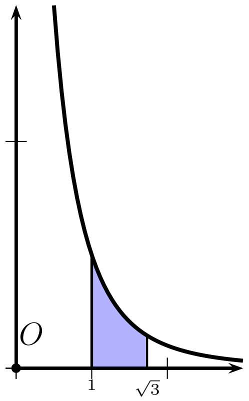
$$S = \int_1^5 \left[(2x - 5) - (x^2 - 4x)\right] dx = \frac{32}{3}$$

18.



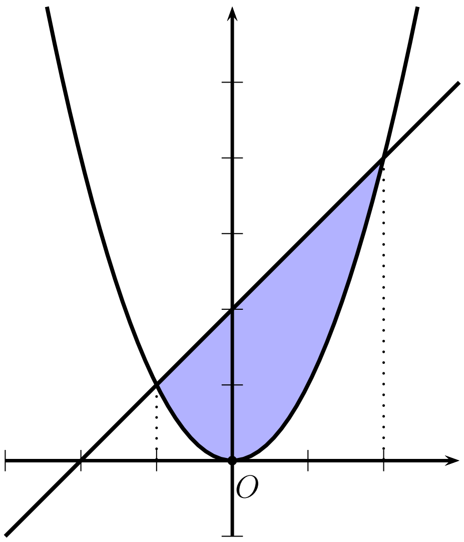
$$S = \int_0^2 (x^3 - 6x^2 + 8x)\, dx + \int_2^4 -(x^3 - 6x^2 + 8x)\, dx =$$
$$= 4 + 4 = 8$$

20.



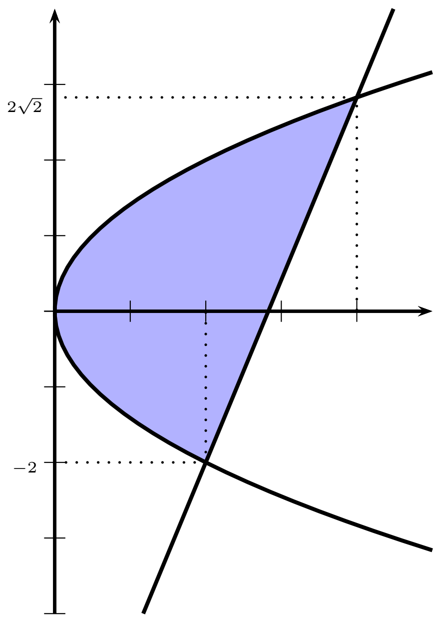
$$S = \int_1^{\sqrt{3}} \frac{1}{x(x^2+1)}\,dx = \frac{1}{2}\ln\left(\frac{3}{2}\right)$$

21.



$$S = \int_{-1}^{2} (x + 2 - x^2)\,dx = \frac{9}{2}$$

22.



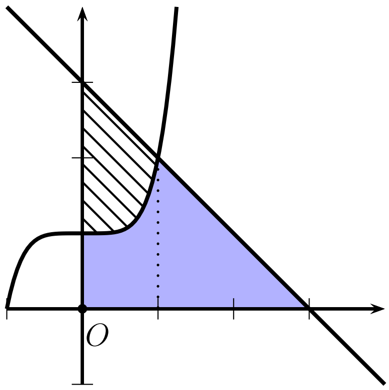
Recta $\equiv y = -2 + (1+\sqrt{2})(x-2)$
Respecto al eje X:

$$S = \int_0^2 2\sqrt{2x}\,dx +$$
$$+ \int_2^4 \left[\sqrt{2x} + 2 - (1+\sqrt{2})(x-2)\right] dx =$$
$$= \left(\frac{16}{3}\right) + \left(\frac{10\sqrt{2}-2}{3}\right) = \frac{10\sqrt{2}+14}{3}$$

Más fácil respecto al eje Y:

$$S = \int_{-2}^{2\sqrt{2}} \left(2 + \frac{y+2}{1+\sqrt{2}} - \frac{y^2}{2}\right) dy =$$
$$= \frac{10\sqrt{2}+14}{3}$$

23.



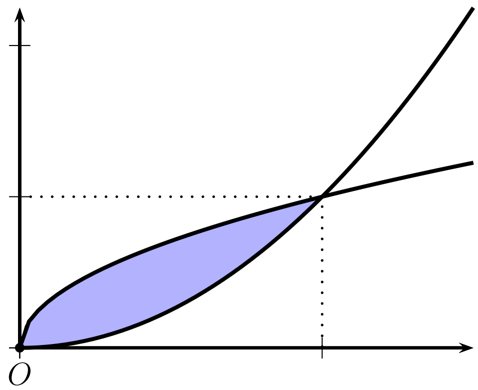
Sea S_1 la superficie de la región rayada:

$$S_1 = \int_0^1 \left[(3-x)-(1+x^5)\right]dx = \frac{4}{3}$$

Sea S_2 la superficie de la región sombreada:

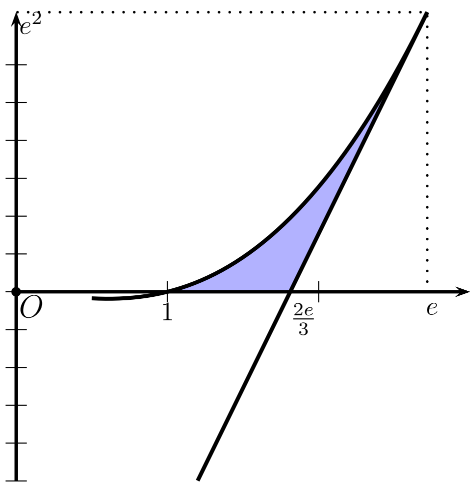
$$S_2 = \frac{9}{2} - \frac{4}{3} = \frac{19}{6}$$

24.



$$S = \int_0^1 (\sqrt{x} - x^2)\,dx = \frac{1}{3}$$

26.



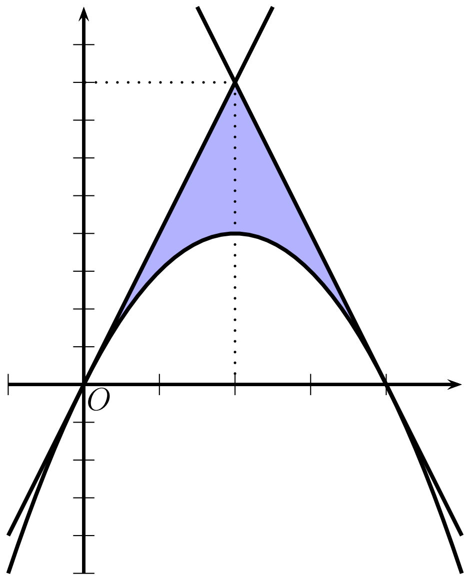
La recta tangente en $x = e$ es:

$$y = 3ex - 2e^2$$

La superficie del triángulo de vértices $(2e/3, 0)$, $(e, 0)$, (e, e^2) es $e^3/6$, luego

$$S = \int_1^e x^2 \ln x\,dx - \frac{e^3}{6} = \frac{e^3+2}{18}$$

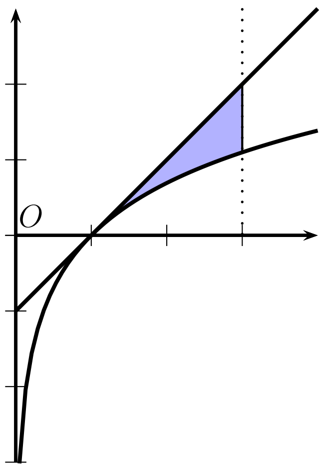
30.



La recta tangente en $x = 0$ es $y = 4x$. La región es simétrica respecto de la recta $x = 2$ (demuéstrelo), luego:

$$S = 2\int_0^2 \left[4x - (4x - x^2)\right] dx = \frac{16}{3}$$

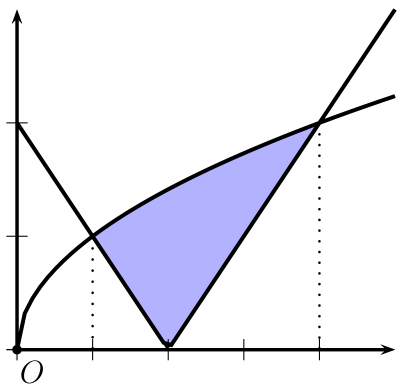
31.



La recta tangente en $x = 1$ es $y = x - 1$.

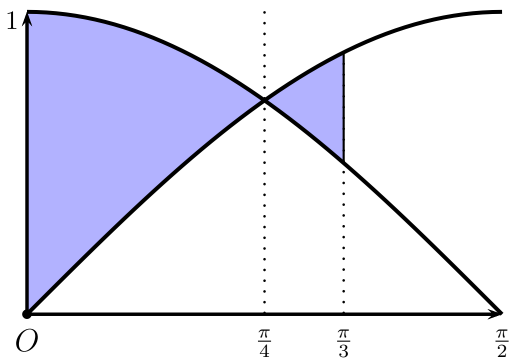
$$S = \int_1^3 \left[(x-1) - \ln x\right] dx = 4 - 3\ln 3$$

32.



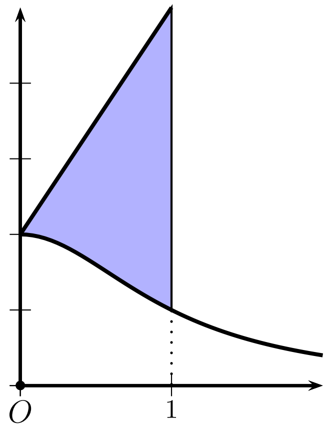
$$S = \int_1^2 \left[\sqrt{x} + (x-2)\right] dx + \\ + \int_2^4 \left[\sqrt{x} - (x-2)\right] dx = \frac{13}{6}$$

39.



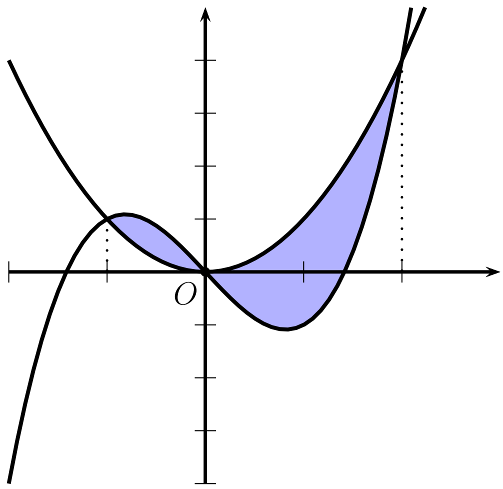
$$
\begin{aligned}
S &= \int_0^{\pi/4} (\cos x - \operatorname{sen} x)\, dx + \\
&+ \int_{\pi/4}^{\pi/3} (\operatorname{sen} x - \cos x)\, dx = \\
&= (\sqrt{2} - 1) + \left(\sqrt{2} - \frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \\
&= 2\sqrt{2} - \frac{3}{2} - \frac{\sqrt{3}}{2}
\end{aligned}
$$

40.



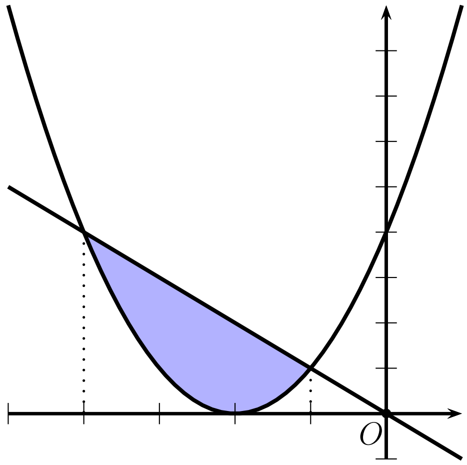
$$S = \int_0^1 \left[(3x+2) - \frac{2}{1+x^2}\right] dx = \frac{7-\pi}{2}$$

44.



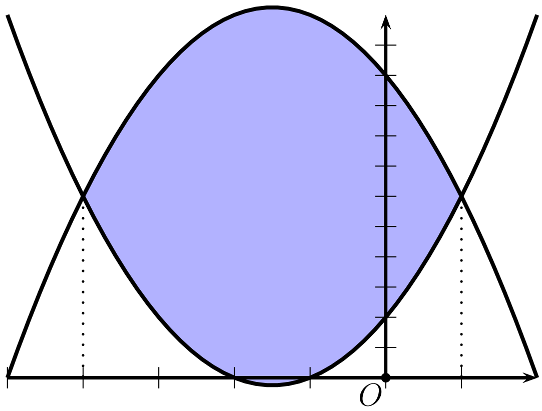
$$
\begin{aligned}
S &= \int_{-1}^{0} \left[(x^3 - 2x) - x^2\right] dx + \\
&+ \int_0^2 \left[x^2 - (x^3 - 2x)\right] dx = \\
&= \frac{5}{12} + \frac{8}{3} = \frac{37}{12}
\end{aligned}
$$

46.



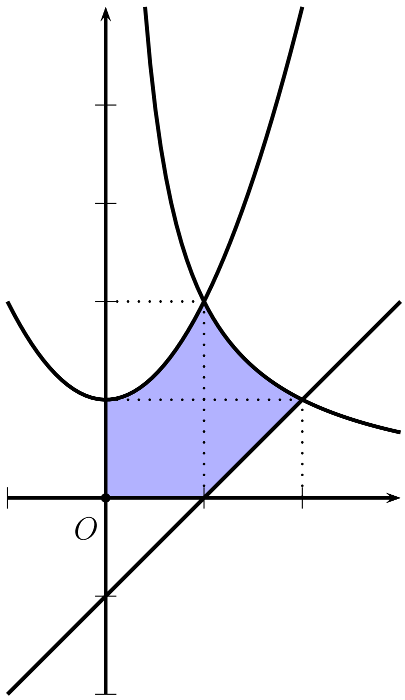
$$S = \int_{-4}^{-1} \left[-x - (x+2)^2\right] dx = \frac{9}{2}$$

47.



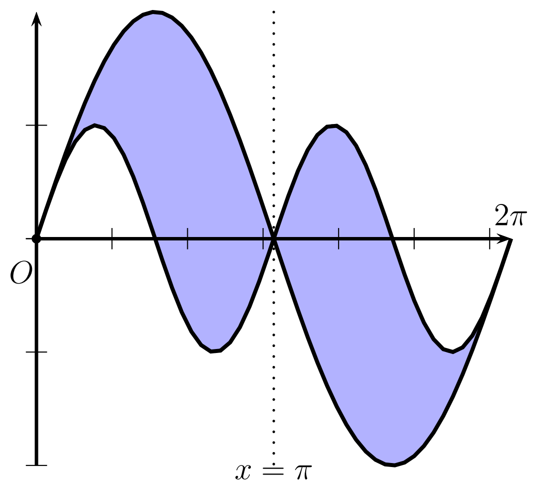
$$S = \int_{-4}^{1} \left[(-x^2 - 3x + 10) - (x^2 + 3x + 2)\right] dx =$$
$$= \frac{125}{3}$$

51.



$$S = \int_{0}^{1} (x^2 + 1)\, dx + \int_{1}^{2} \left[\frac{2}{x} - (x-1)\right] dx =$$
$$= \frac{4}{3} + \left(-\frac{1}{2} + 2\ln 2\right) = \frac{5}{6} + 2\ln 2$$

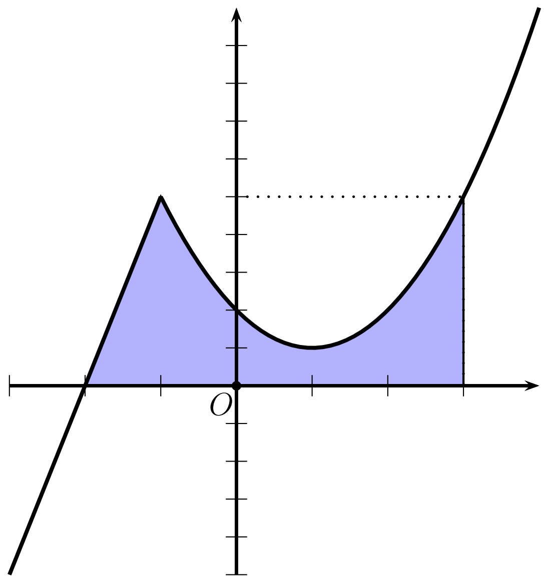
55.



Por simetría:

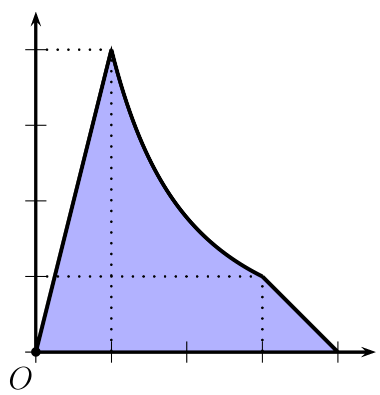
$$S = 2\int_0^{\pi} \big(2\,\mathrm{sen}\,x - \mathrm{sen}\,2x\big)\,dx = 8$$

57.



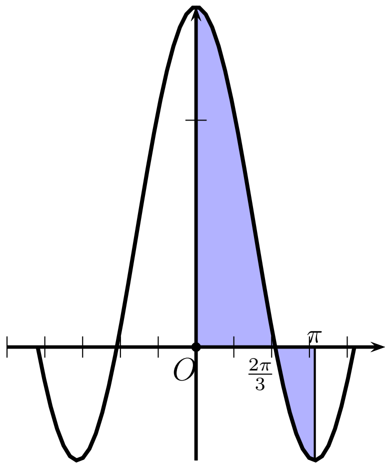
$$S = \int_{-2}^{-1} (5x+10)\,dx + \int_{-1}^{3} (x^2 - 2x + 2)\,dx =$$
$$= \frac{5}{2} + \frac{28}{3} = \frac{71}{6}$$

62.



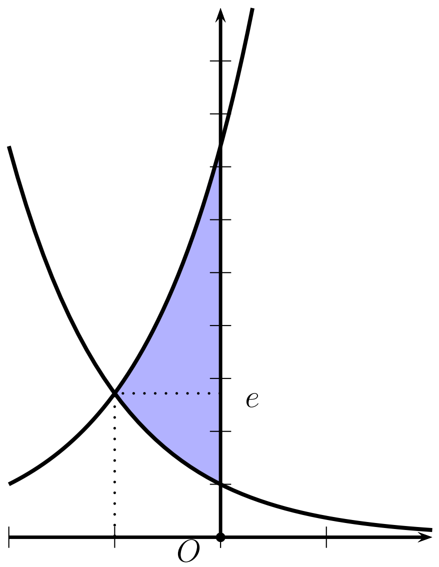
$$S = \int_0^1 4x\,dx + \int_1^3 \frac{16}{(x+1)^2}\,dx + \int_3^4 (4-x)\,dx =$$
$$= 2 + 4 + \frac{1}{2} = \frac{13}{2}$$

63.



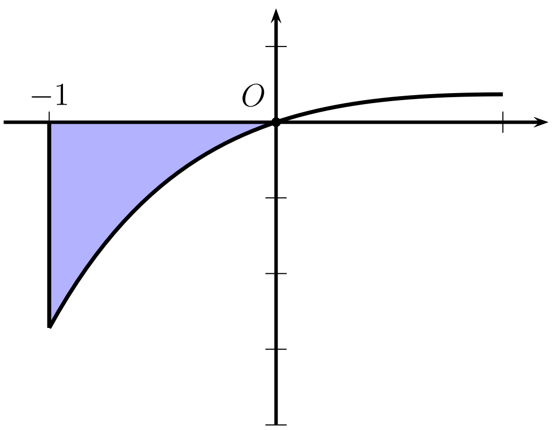
$$S = \int_0^{2\pi/3} \left(\frac{1}{2} + \cos x\right)\, dx +$$
$$+ \int_{2\pi/3}^{\pi} - \left(\frac{1}{2} + \cos x\right)\, dx =$$
$$= \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) + \left(-\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right) =$$
$$= \sqrt{3} + \frac{\pi}{6}$$

64.



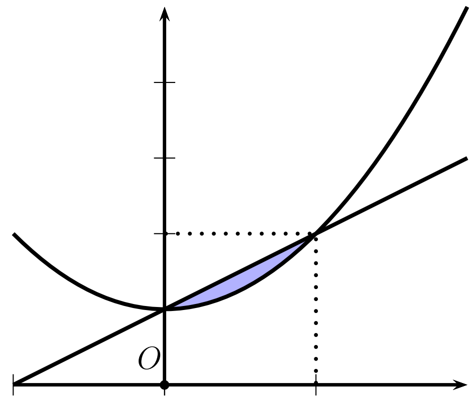
$$S = \int_{-1}^{0} \left(e^{x+2} - e^{-x}\right) dx = (e-1)^2$$

68.



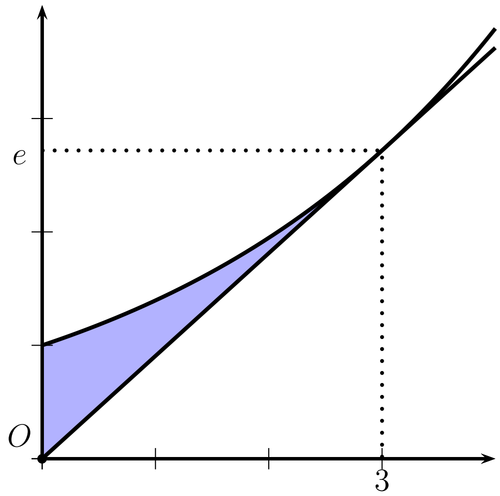
$$S = \int_{-1}^{0} -xe^{-x}\, dx = 1$$

71.



$$S = \int_0^1 \left[(1+x) - (1+x^2)\right] dx = \frac{1}{6}$$

72.

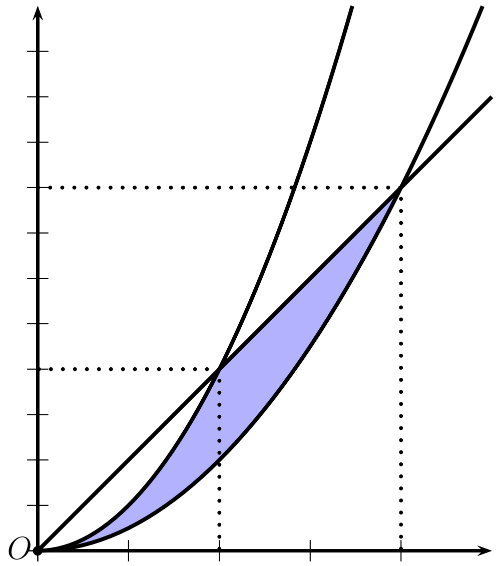


La recta tangente en $x = 3$ es:

$$y = \frac{ex}{3}$$

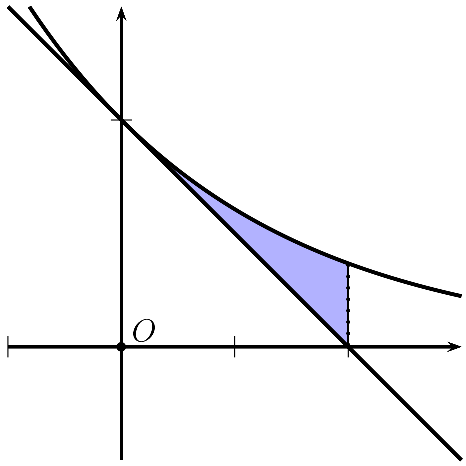
$$S = \int_0^3 \left(e^{x/3} - \frac{ex}{3}\right) dx = \frac{3e}{2} - 3$$

74.



$$S = \int_0^2 \left(x^2 - \frac{x^2}{2}\right) dx + \int_2^4 \left(2x - \frac{x^2}{2}\right) dx =$$
$$= \frac{4}{3} + \frac{8}{3} = 4$$

77.

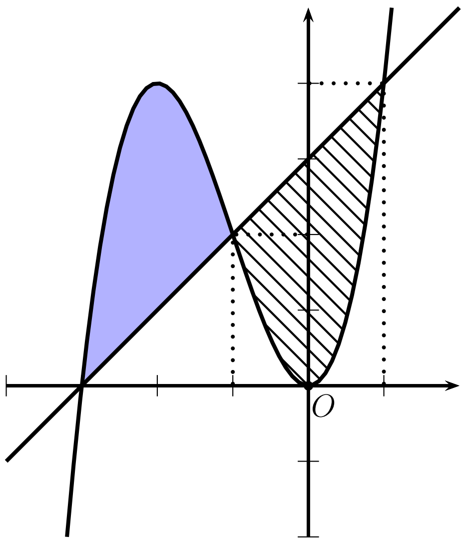


La recta tangente en $x = 0$ es:

$$y = 1 - \frac{x}{2}$$

$$S = \int_0^2 \left[e^{-x} - \left(1 - \frac{x}{2}\right) \right] dx = 1 - \frac{2}{e}$$

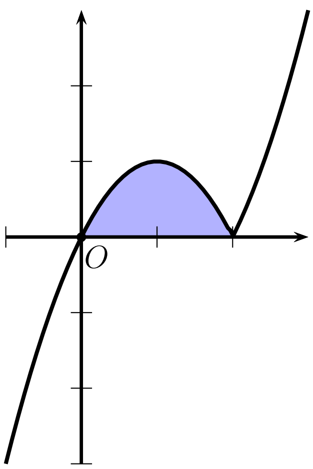
82.



$$S_{\text{sombreada}} = \int_{-3}^{-1} \left[(x^3 + 3x^2) - (x + 3) \right] dx = 4$$

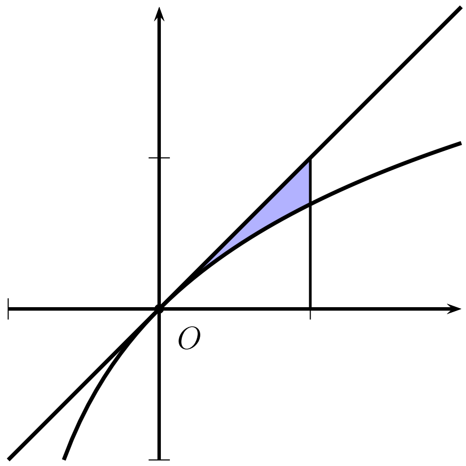
$$S_{\text{rayada}} = \int_{-1}^{1} \left[(x + 3) - (x^3 + 3x^2) \right] dx = 4$$

83.



$$S = \int_0^2 x|x-2| \, dx = \int_0^2 -x(x-2) \, dx = \frac{4}{3}$$

84.

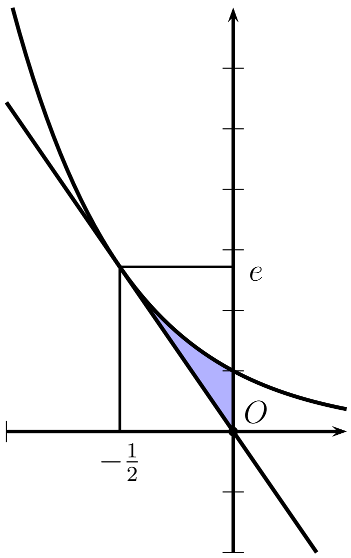


La recta tangente en $x = 0$ es:

$$y = x$$

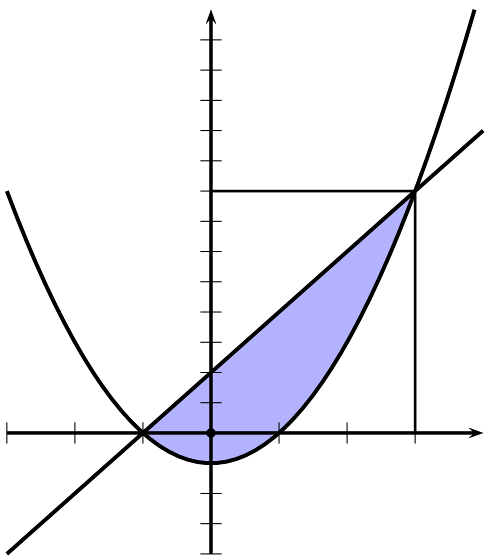
$$S = \int_0^1 \big[x - \ln(1+x)\big]\, dx = \frac{3}{2} - 2\ln 2$$

86.



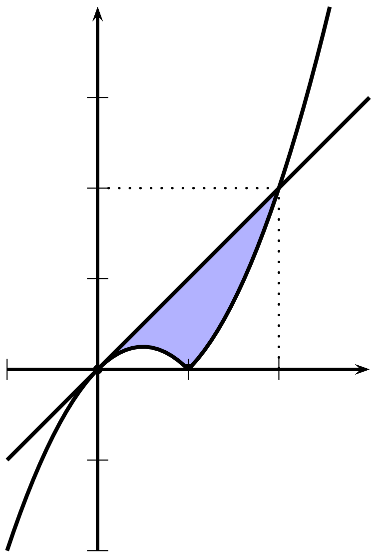
$$S = \int_{-\frac{1}{2}}^{0} \left(e^{-2x} + 2ex\right) dx = \frac{e}{4} - \frac{1}{2}$$

88.



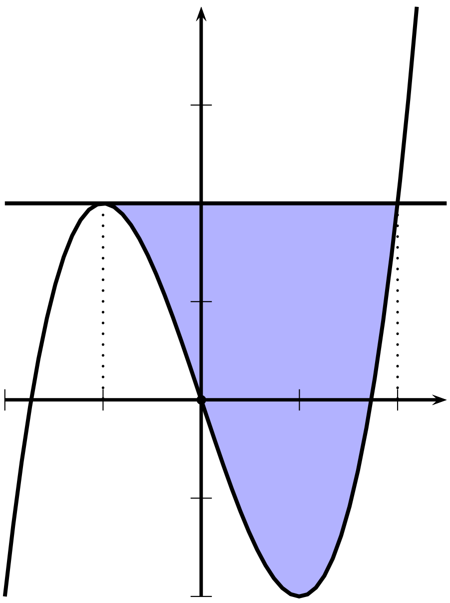
$$S = \int_{-1}^{3} \big[(2x+2) - (x^2-1)\big]\, dx = \frac{32}{3}$$

89.



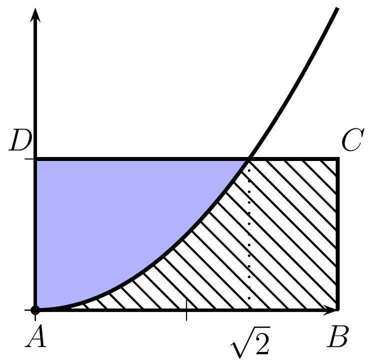
$$S = \int_0^2 \big(x - x|x-1|\big)\,dx =$$
$$= \int_0^1 \big[x + x(x-1)\big]\,dx + \int_1^2 \big[x - x(x-1)\big]\,dx =$$
$$= \frac{1}{3} + \frac{2}{3} = 1$$

90.



$$S = \int_{-1}^{2} \big[2 - (x^3 - 3x)\big]\,dx = \frac{27}{4}$$

91.

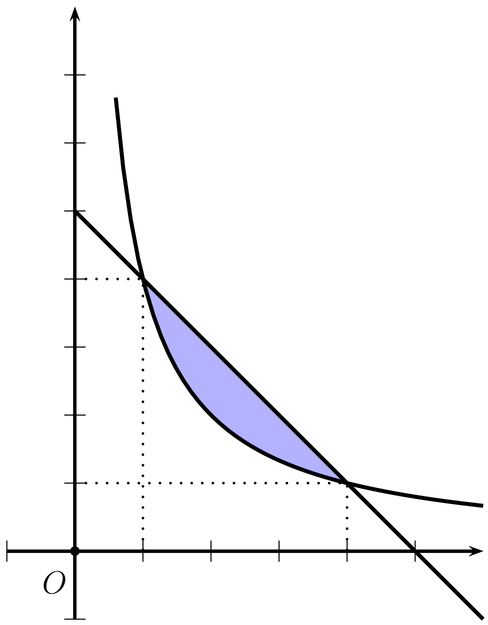


$$S_{\text{sombreada}} = \int_0^{\sqrt{2}} \left(1 - \frac{x^2}{2}\right)\,dx = \frac{2\sqrt{2}}{3}$$

La superficie del rectángulo $ABCD$ es 2, luego

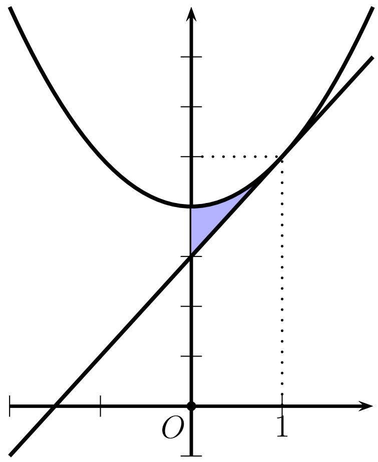
$$S_{\text{rayada}} = 2 - \frac{2\sqrt{2}}{3} = \frac{6 - 2\sqrt{2}}{3}$$

93.



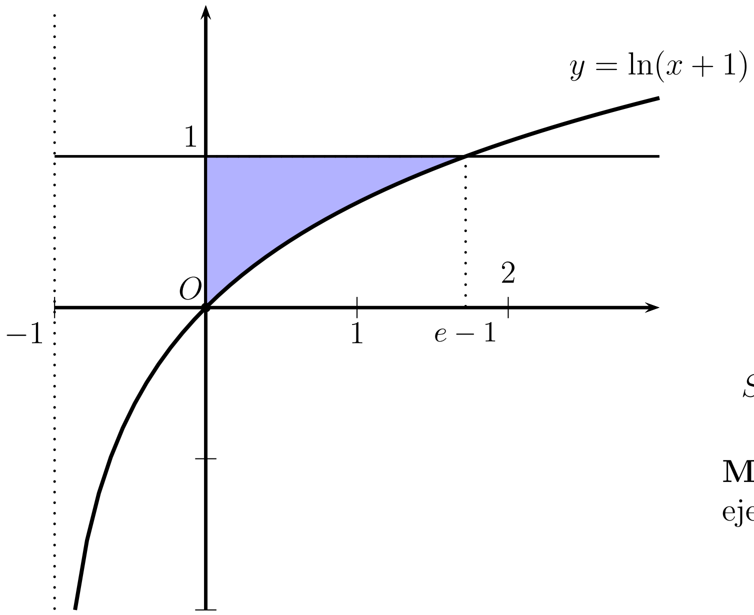
$$S = \int_1^4 \left(5 - x - \frac{4}{x}\right)\, dx = \frac{15}{2} - 8\ln 2$$

94.



$$S = \int_0^1 \left[x^2 + 4 - (2x+3)\right]\, dx = \frac{1}{3}$$

95.

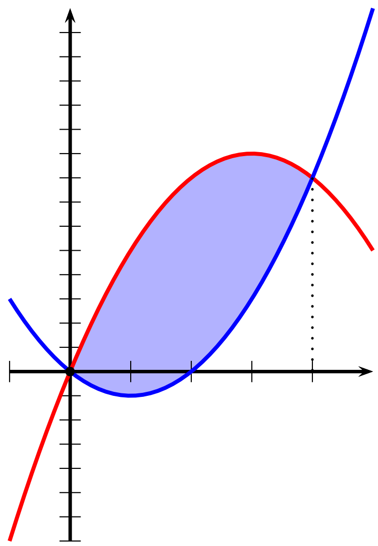


$$S=\int_0^{e-1}\left[1-\ln(x+1)\right]\,dx=e-2$$

Muchísimo más fácil con respecto al eje Y:

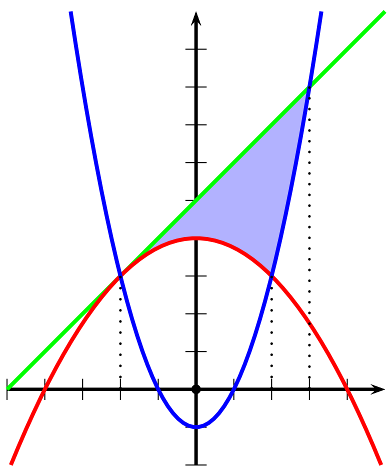
$$S=\int_0^1 (e^y-1)\,dy=e-2$$

96.



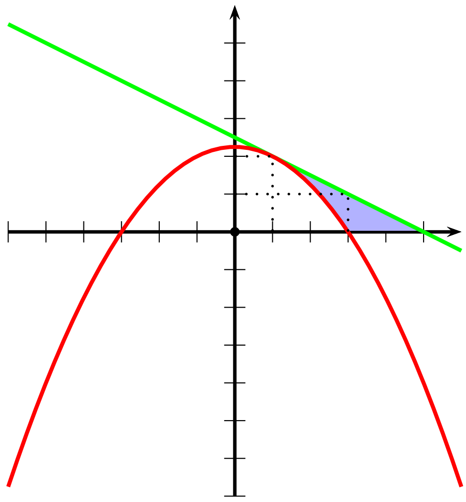
$$S=\int_0^4 \left[(6x-x^2)-(x^2-2x)\right]\,dx=\frac{64}{3}$$

97.



$$S=S_1+S_2=\int_{-2}^{2}\left[(x+5)-\left(4-\frac{x^2}{4}\right)\right]\,dx+$$
$$\int_2^3\left[(x+5)-(x^2-1)\right]\,dx=\frac{16}{3}+\frac{13}{6}=\frac{15}{2}$$

101.



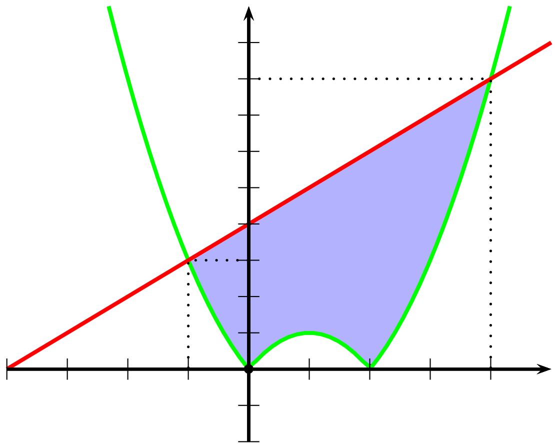
Integrando respecto al eje X:

$$S = S_1 + S_2 = \int_1^3 \left(\frac{5-x}{2} - \frac{9-x^2}{4} \right) dx + \int_3^5 \frac{5-x}{2}\, dx = \frac{2}{3} + 1 = \frac{5}{3}$$

Integrando respecto al eje Y:

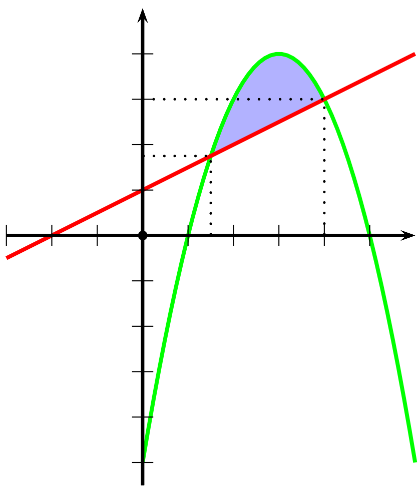
$$S = \int_0^2 \left[(5-2y) - \sqrt{9-4y} \right] dy = \frac{5}{3}$$

102.



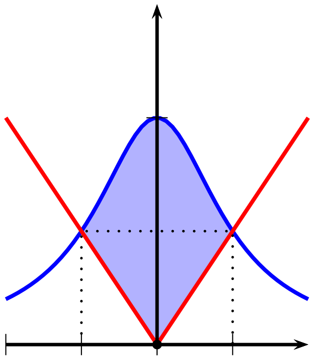
$$S = \int_{-1}^4 \left[x + 4 - |x \cdot (x-2)| \right] dx =$$
$$= \int_{-1}^0 \left[x + 4 - x(x-2) \right] dx +$$
$$+ \int_0^2 \left[x + 4 + x(x-2) \right] dx +$$
$$+ \int_2^4 \left[x + 4 - x(x-2) \right] dx =$$
$$= \frac{13}{6} + \frac{26}{3} + \frac{22}{3} = \frac{109}{6}$$

103.



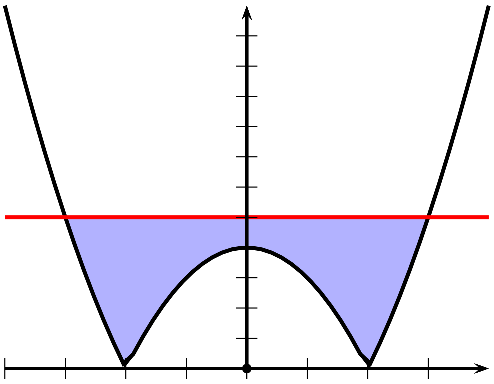
$$S = \int_{3/2}^4 \left(-x^2 + 6x - 5 - \frac{x+2}{2} \right) dx =$$
$$= \frac{125}{48}$$

104.



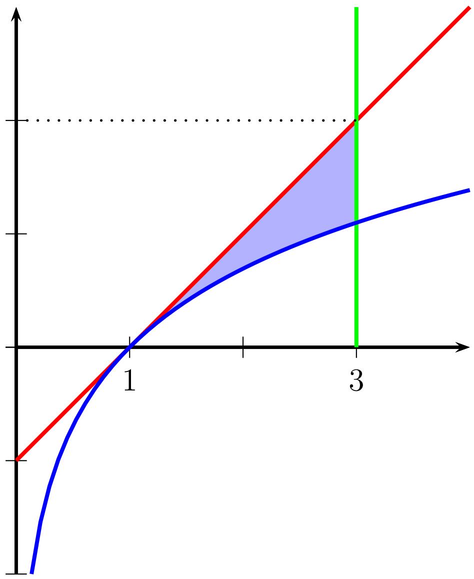
$$S = 2\int_0^1 \left(\frac{1}{1+x^2} - \frac{x}{2}\right)\,dx =$$
$$= \frac{\pi - 1}{2}$$

109.



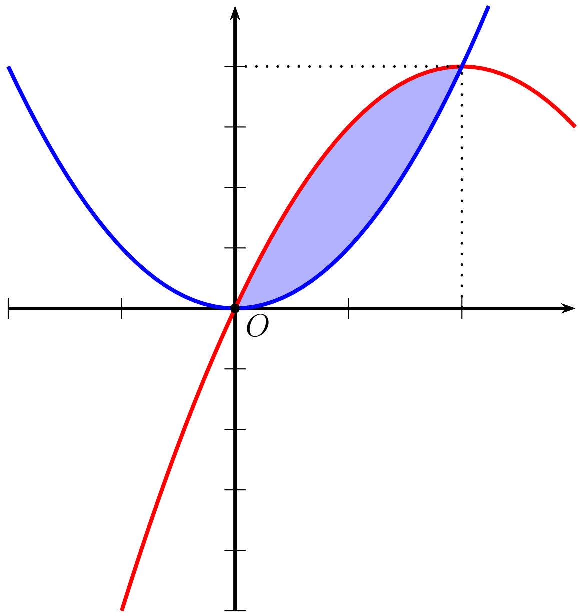
Sea $f(x) = |x^2 - 4|$. Entonces: $S = 2(I_1 + I_2)$, $I_1 = \int_0^2 \big(5 - f(x)\big)\,dx = \frac{14}{3}$, $I_2 = \int_2^3 \big(5 - f(x)\big)\,dx = \frac{8}{3}$, y de aquí $S = \frac{44}{3}$.

110.



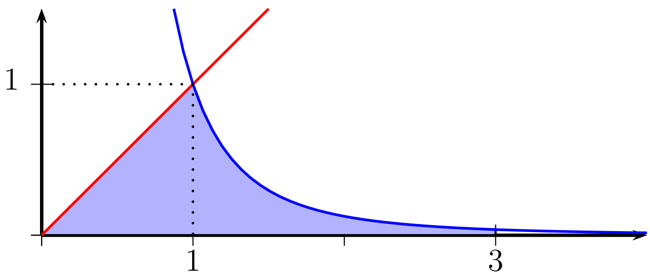
$$S = \int_1^3 (x - 1 - \ln x)\,dx = 4 - 3\ln 3$$

112.



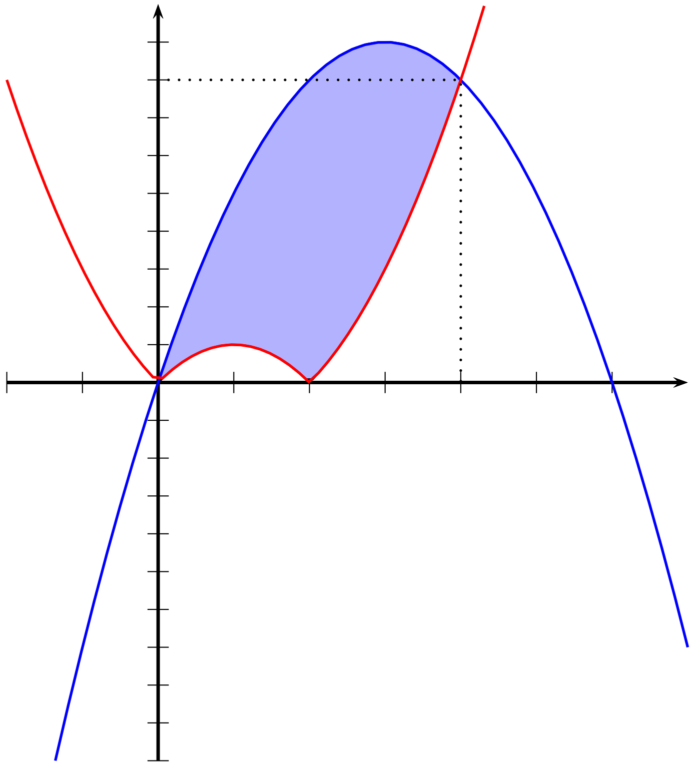
$$S = \int_0^2 \left[(-x^2 + 4x) - x^2\right] \, dx = \frac{8}{3}$$

114.



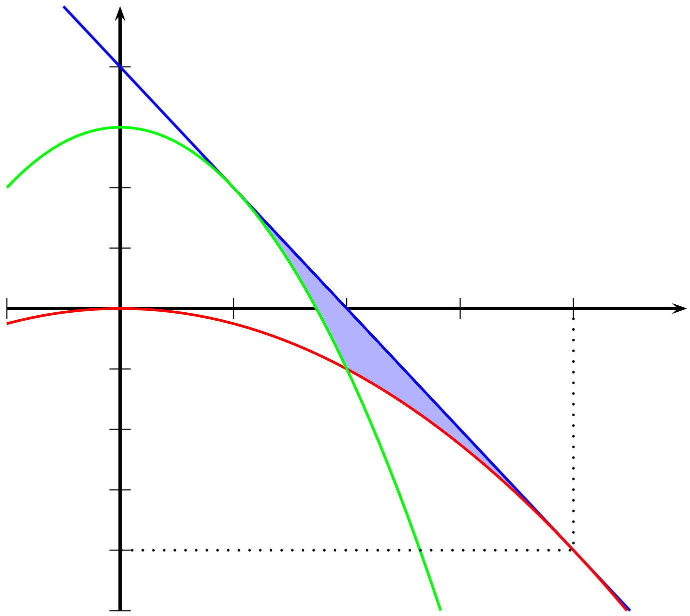
$$S = \int_0^1 x \, dx + \int_1^3 \frac{1}{x^3} \, dx = \frac{17}{18}$$

115.



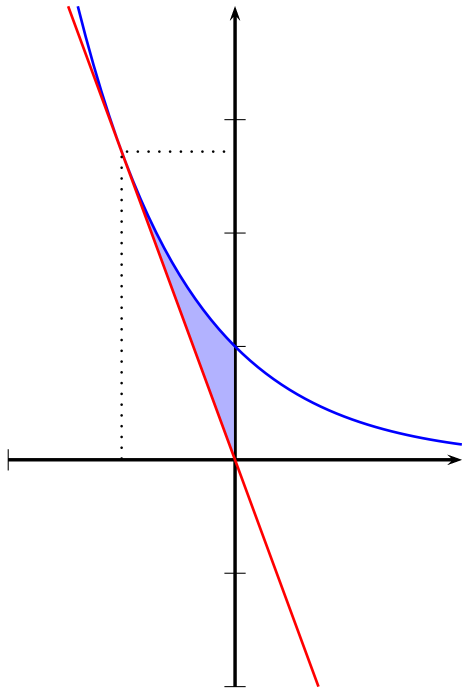
$$S=\int_0^4\left[(6x-x^2)-|x^2-2x|\right]\,dx=$$
$$=\int_0^2\left[(6x-x^2)-(2x-x^2)\right]\,dx+$$
$$\int_2^4\left[(6x-x^2)-(x^2-2x)\right]\,dx=$$
$$=8+\frac{32}{3}=\frac{56}{3}$$

116.



$$S=\int_1^2\left[(4-2x)-(3-x^2)\right]\,dx+$$
$$\int_2^4\left[(4-2x)-\left(-\frac{x^2}{4}\right)\right]\,dx=$$
$$=\frac{1}{3}+\frac{2}{3}=1$$

118.



$$S = \int_{-\frac{1}{2}}^{0} (e^{-2x} + 2ex)\,dx =$$

$$= \left[-\frac{1}{2}e^{-2x} + ex^2\right]_{-\frac{1}{2}}^{0} = \frac{e-2}{4}$$