

# Soluciones a algunos problemas gráficos correspondientes a los problemas de Matemáticas II para 2º de Bachillerato

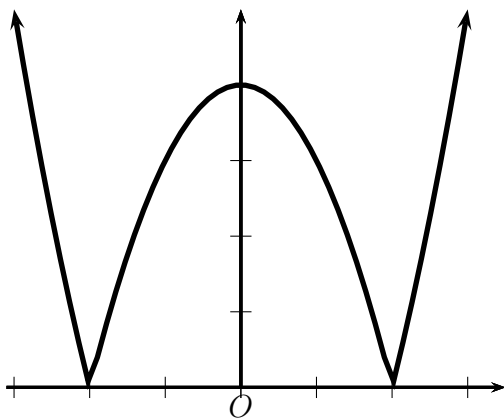
Pedro González Ruiz  
Catedrático de Matemáticas

Versión 18: junio de 2021

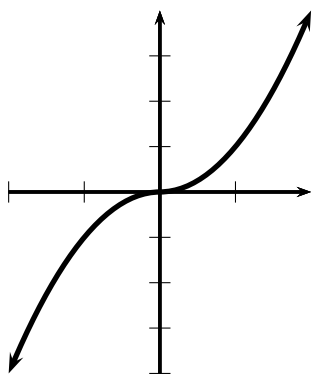
# Capítulo 2

## Sección 2.2

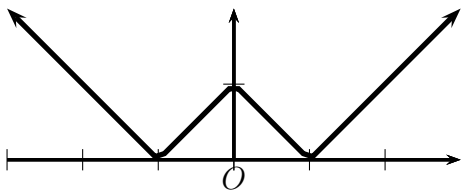
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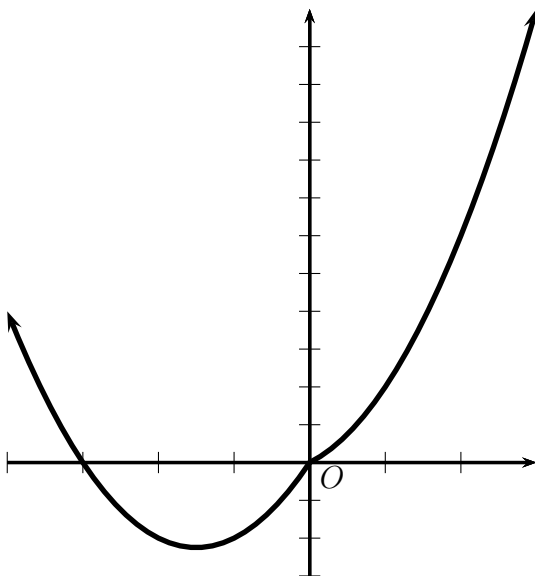
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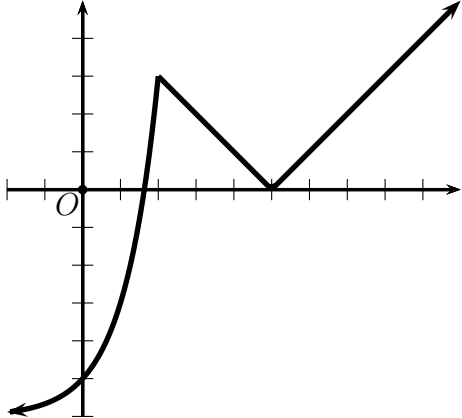


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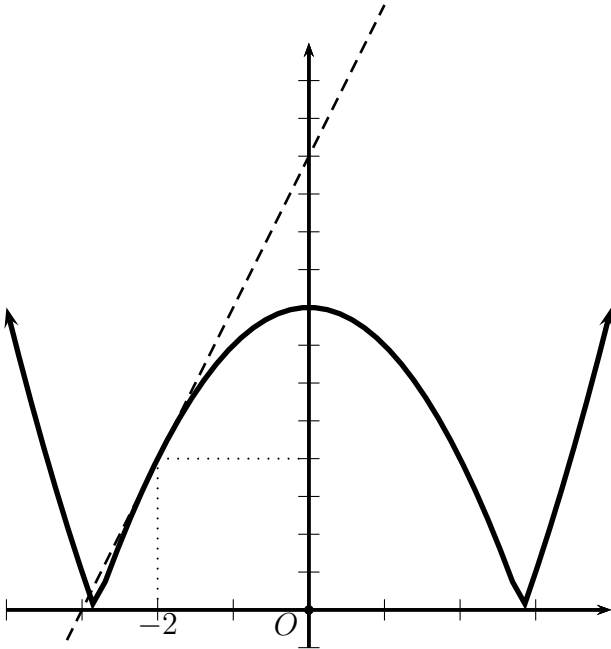


Sección 2.8

7.

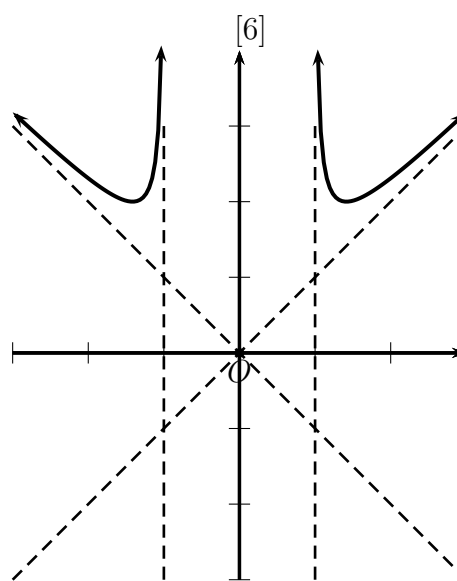
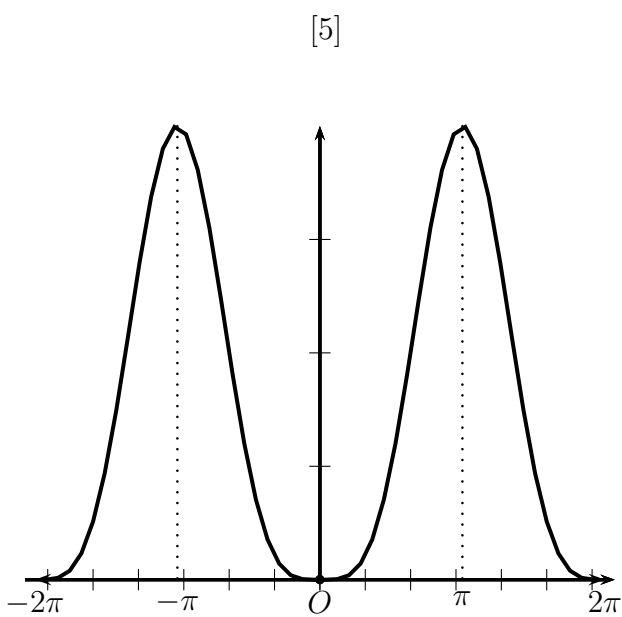
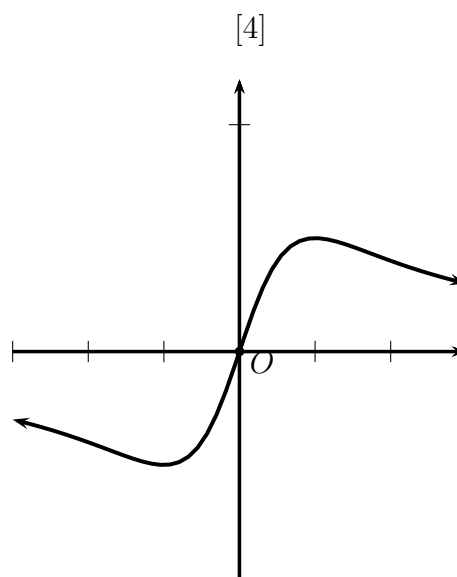
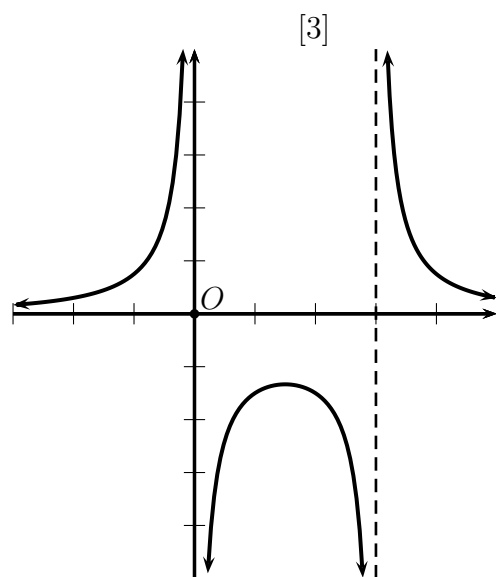
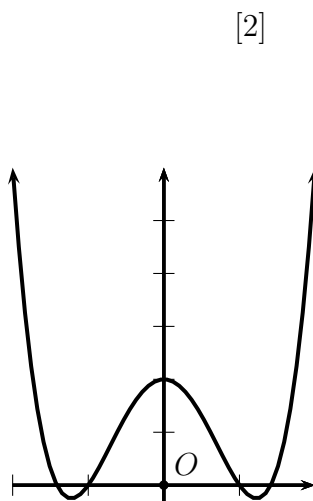
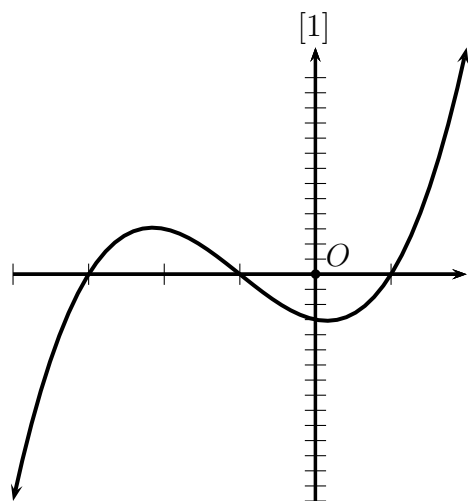


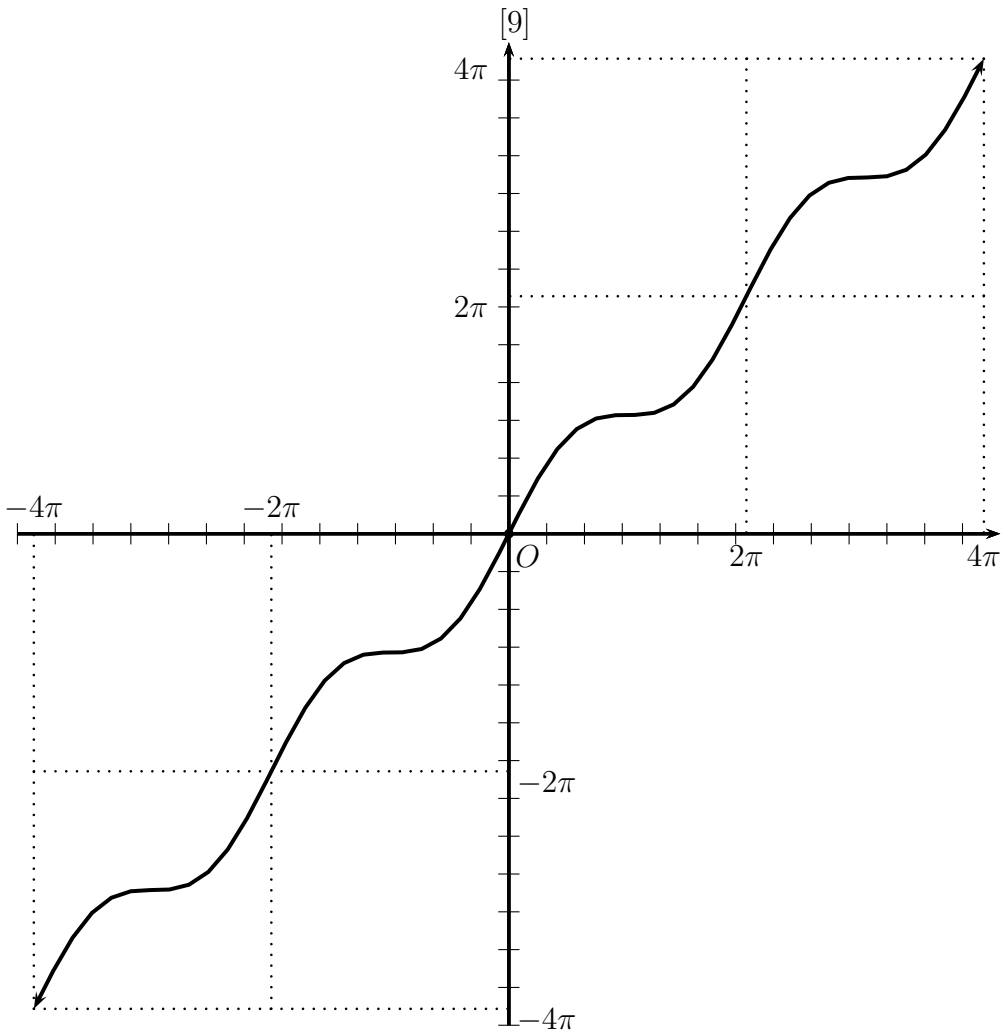
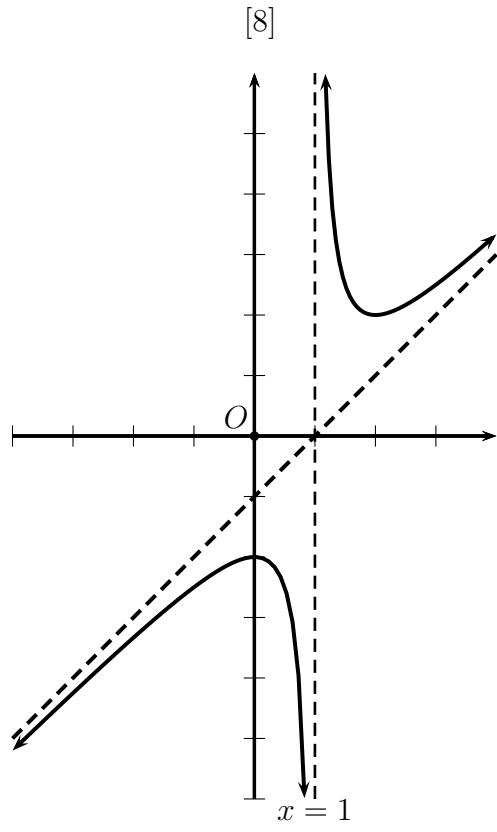
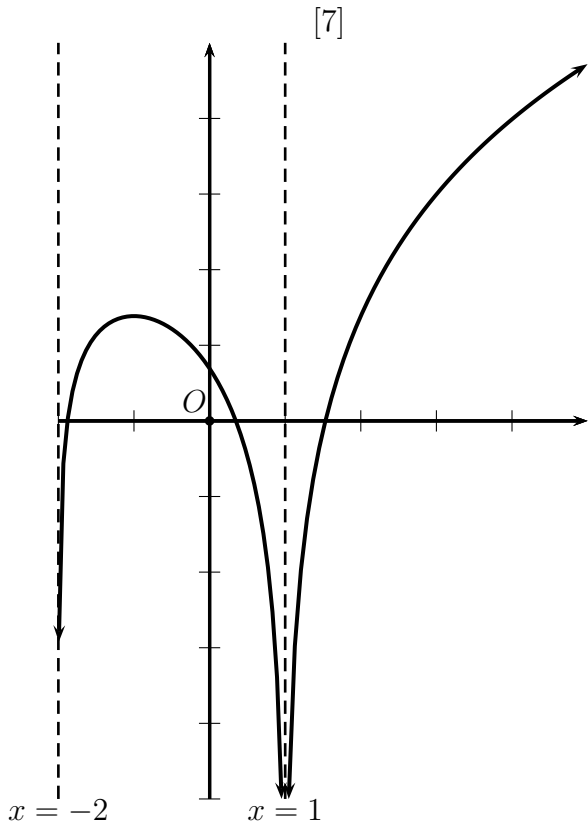
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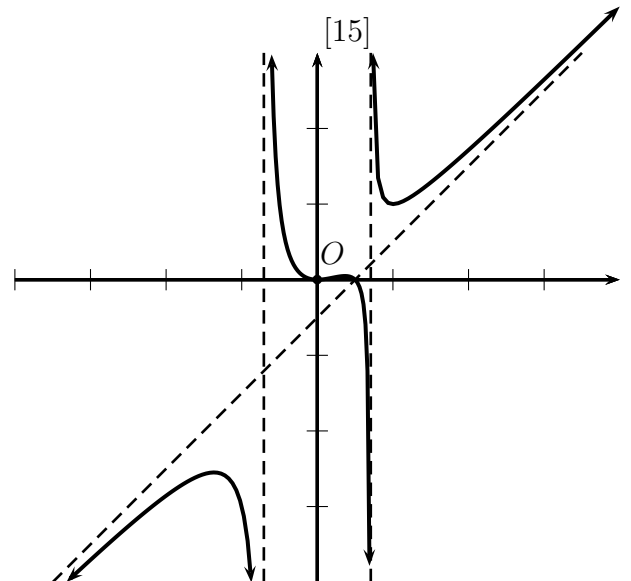
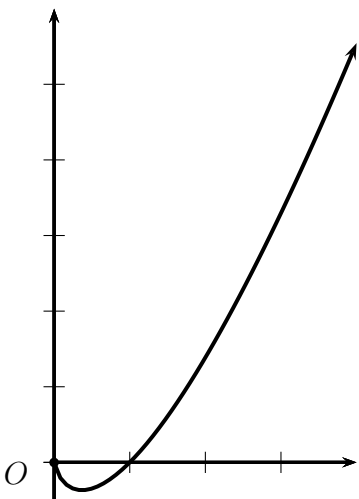
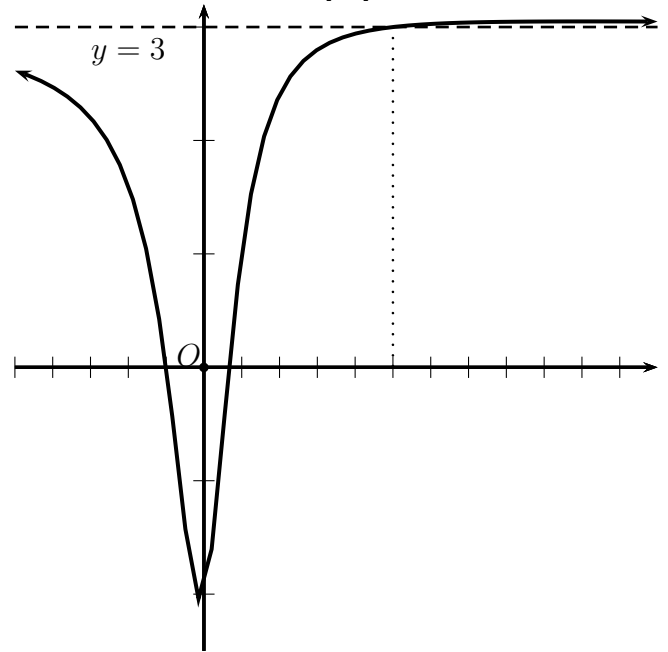
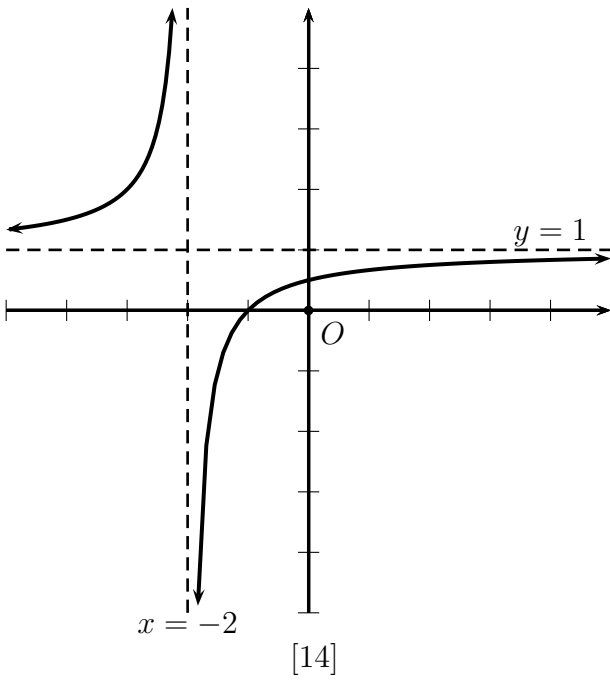
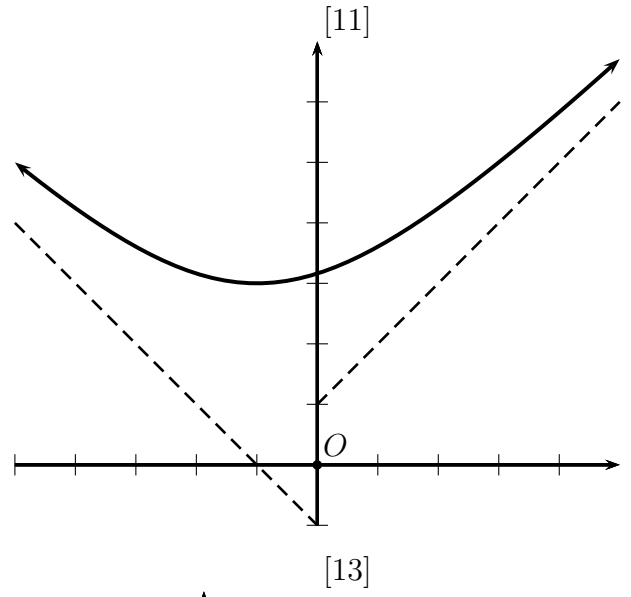
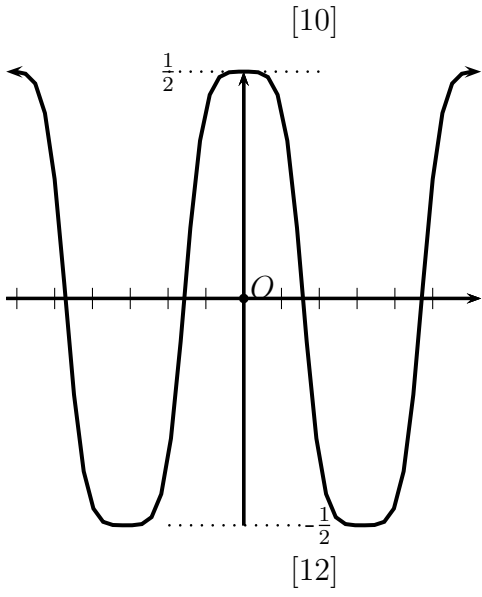


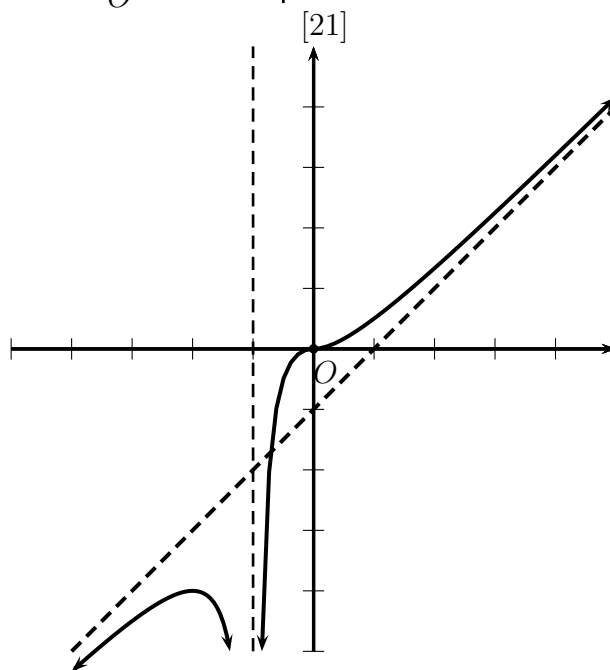
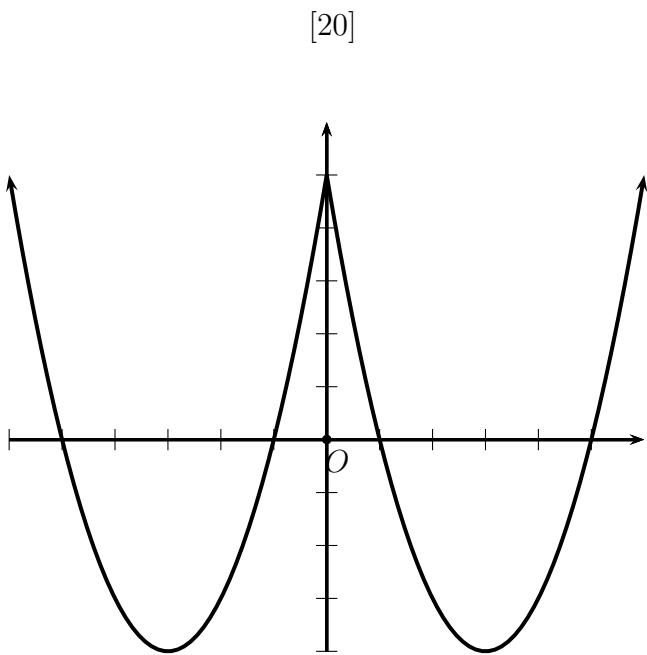
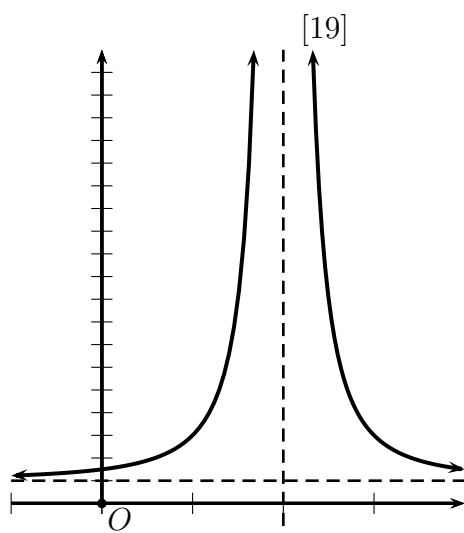
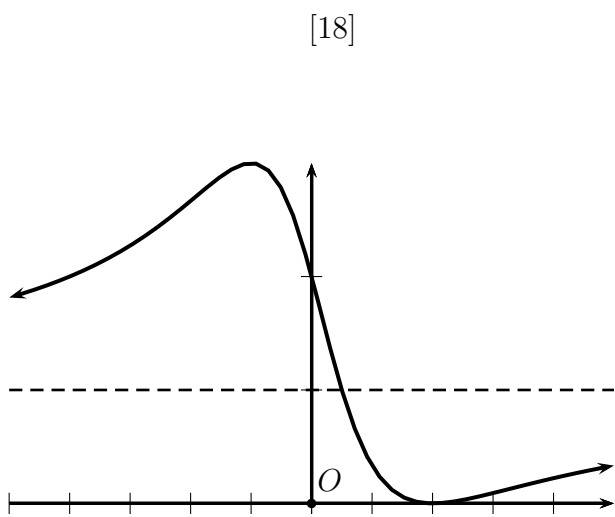
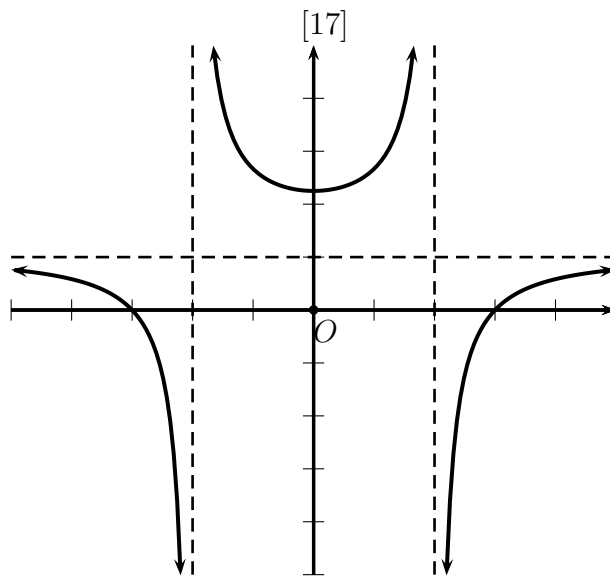
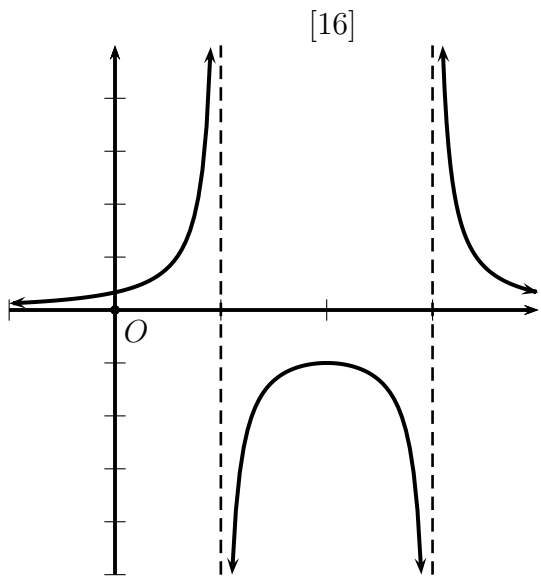
# Capítulo 4

## Sección 4.1

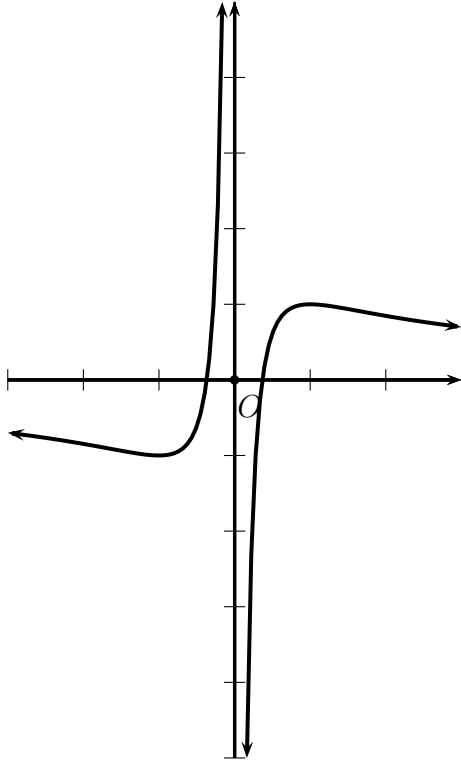




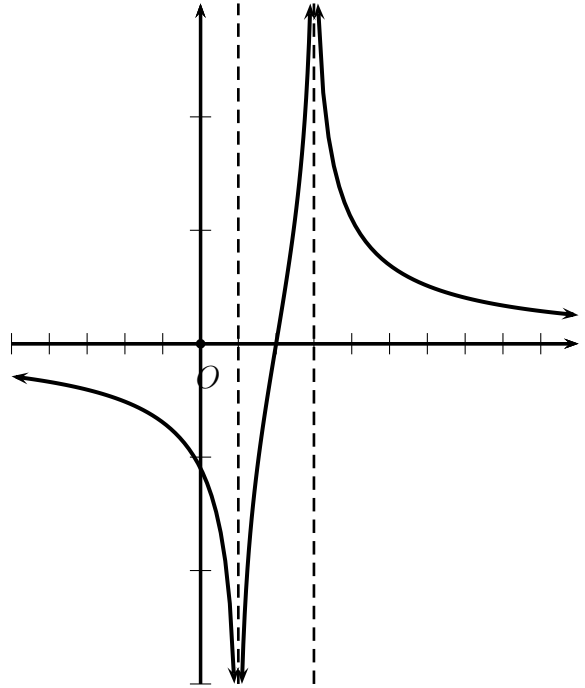




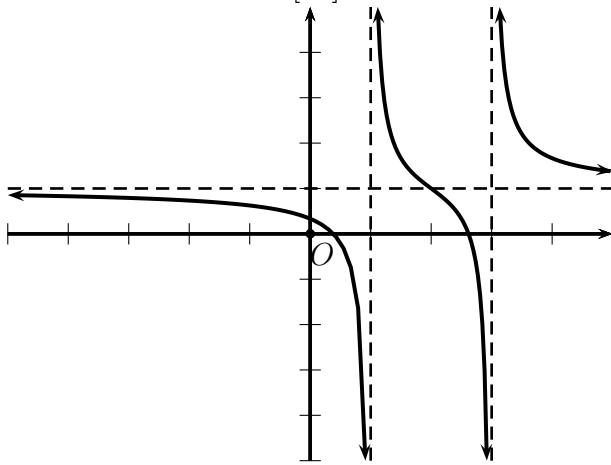
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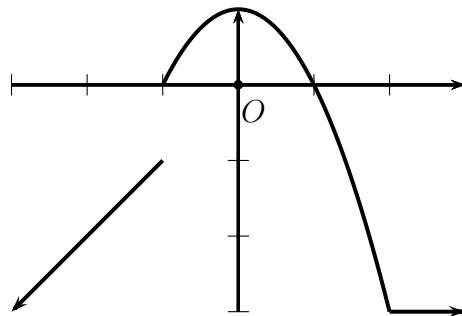
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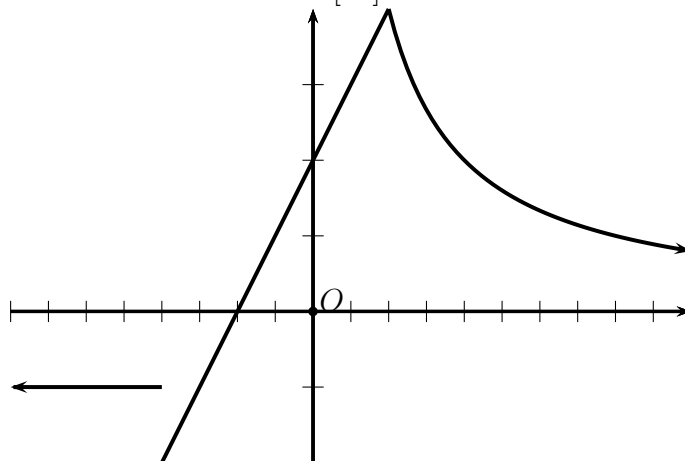
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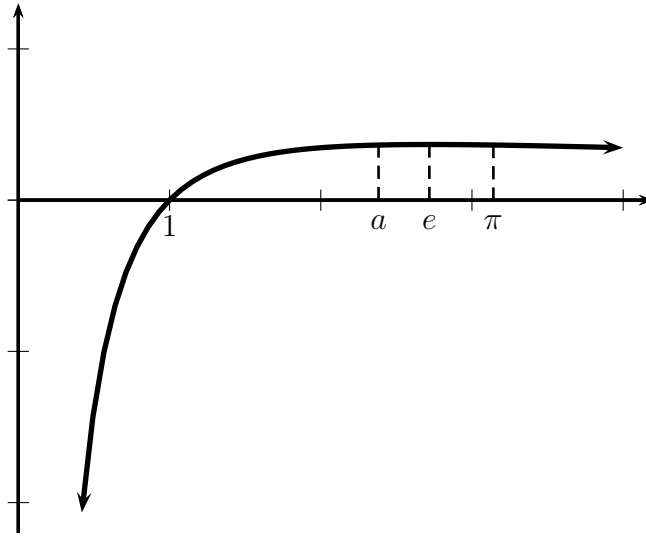
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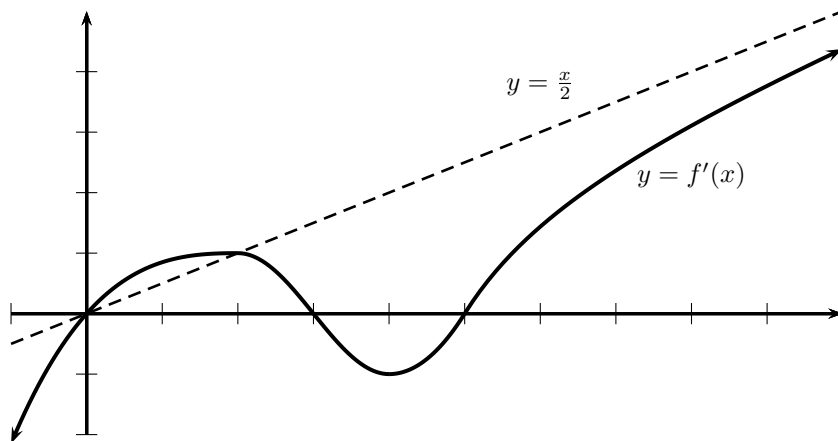


## Sección 4.2

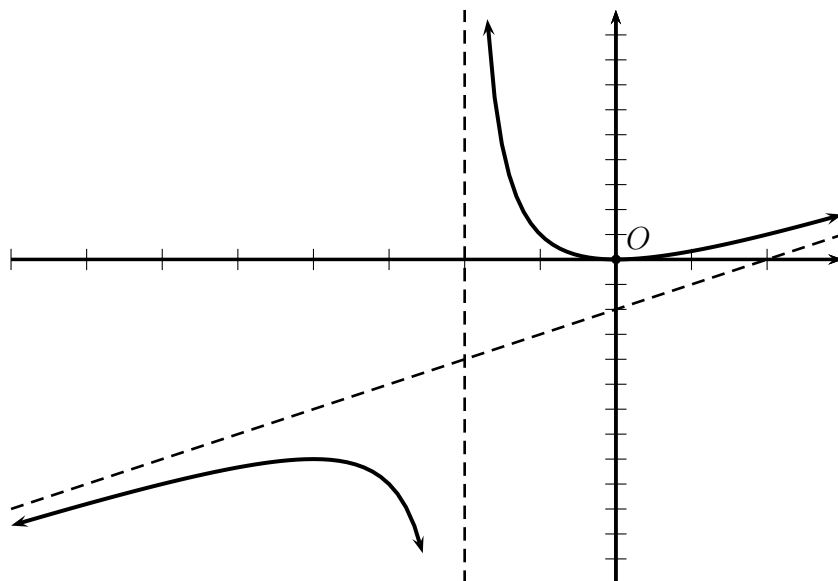
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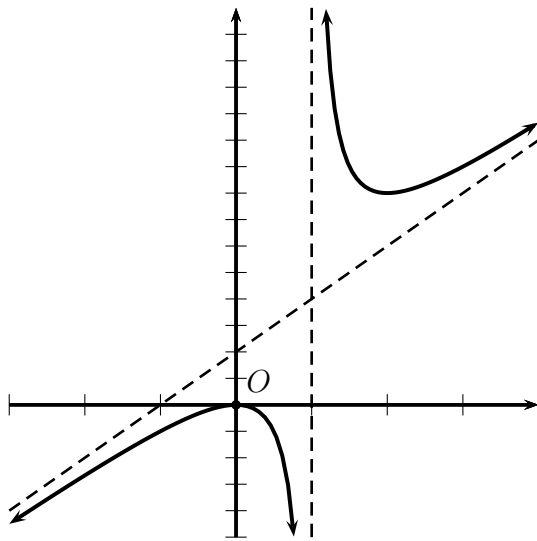
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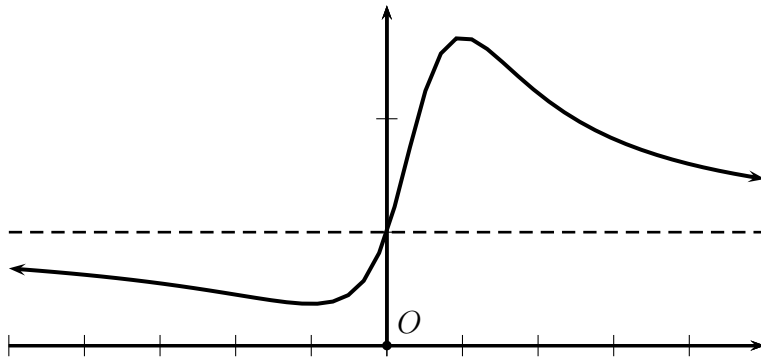
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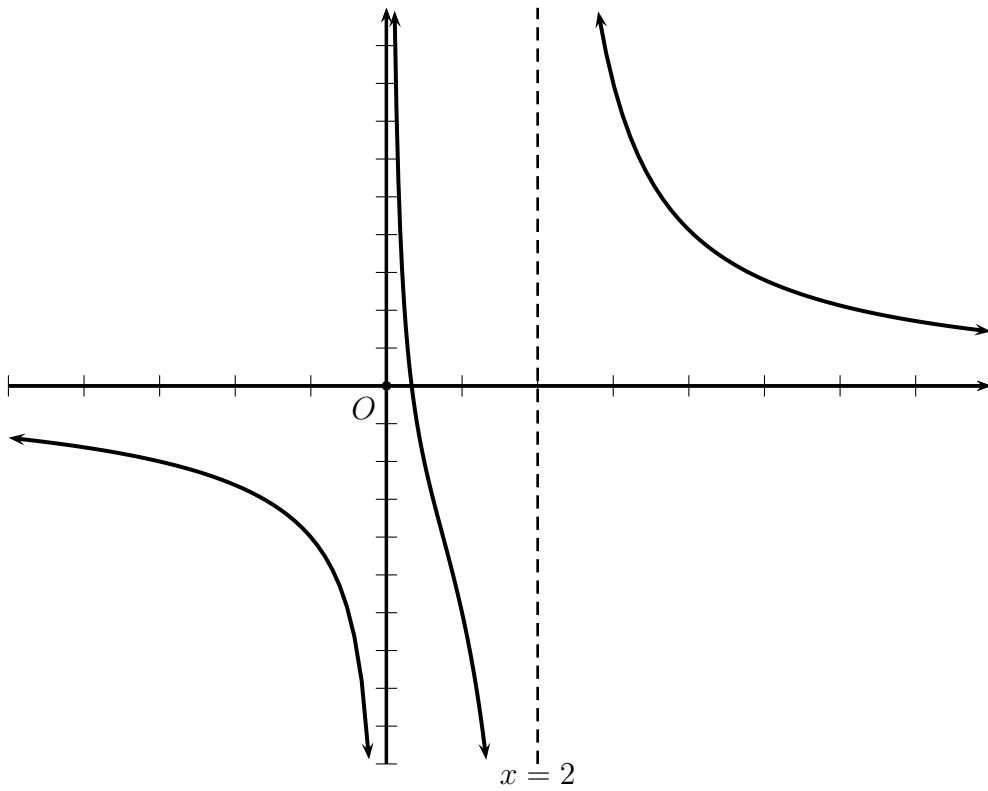
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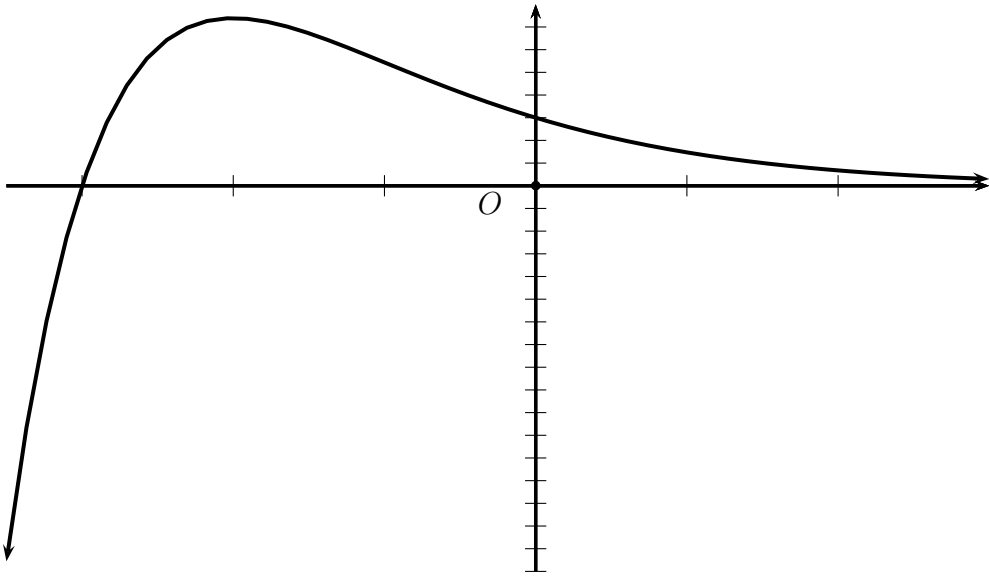
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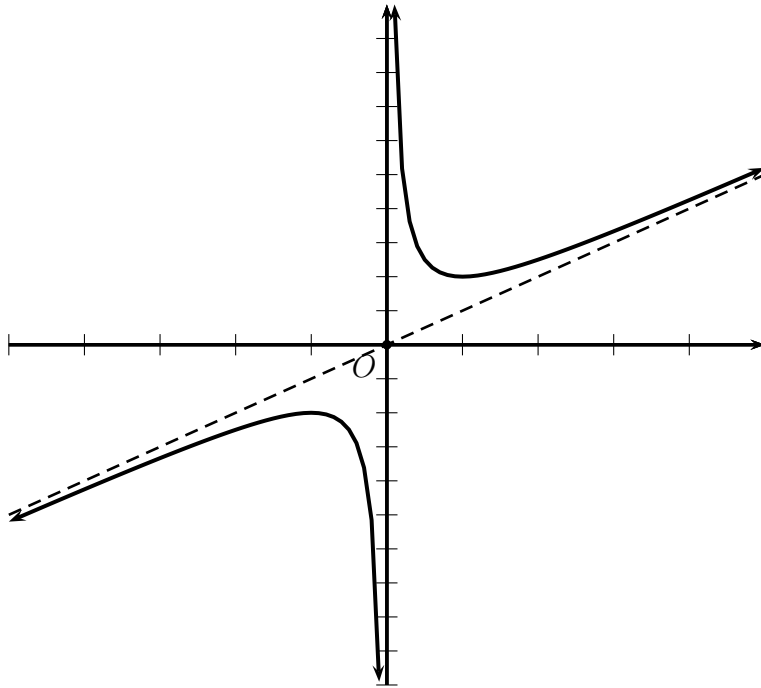
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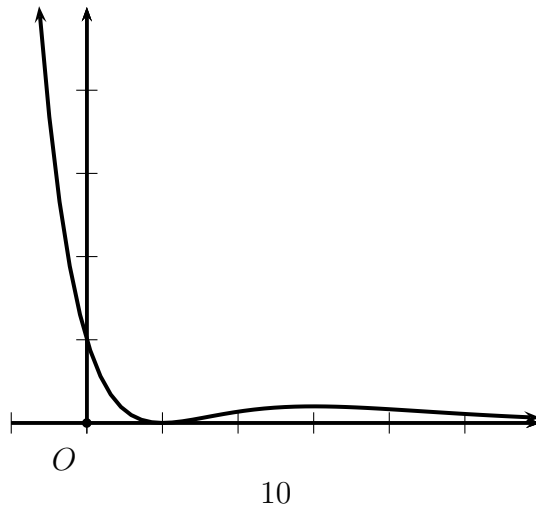
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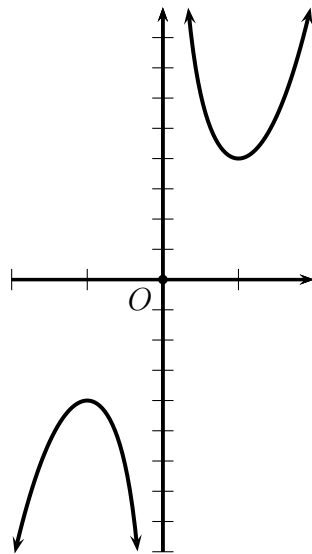
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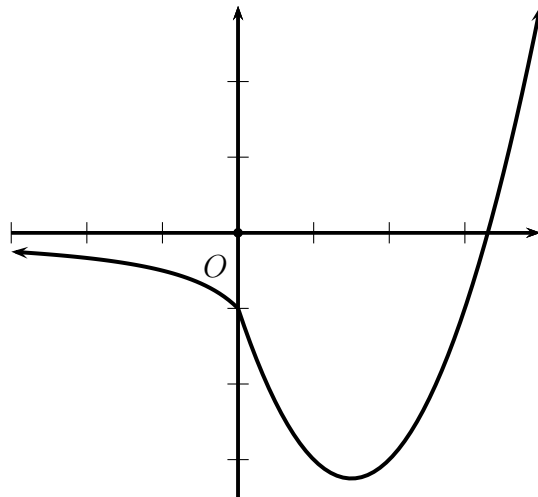
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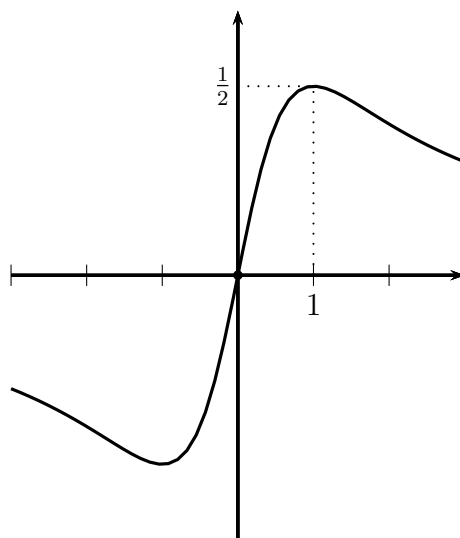
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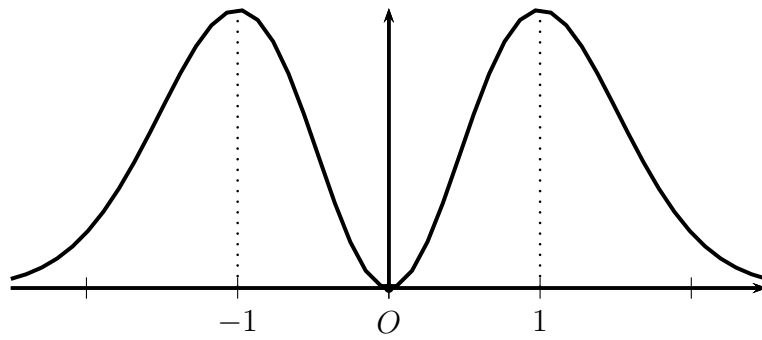
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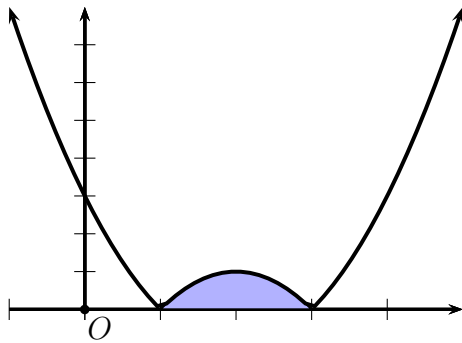


21.



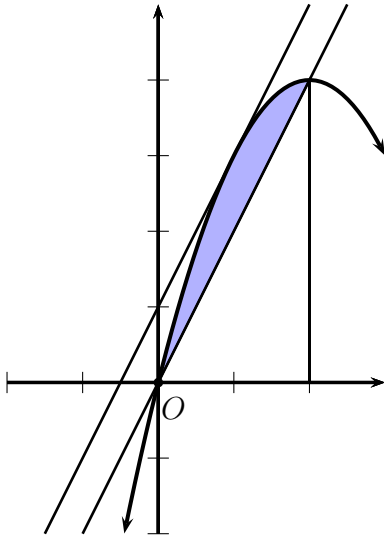
## Capítulo 6

3.



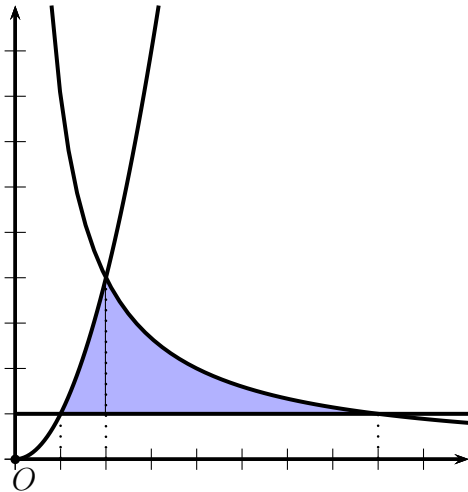
$$\begin{aligned}
 S &= \int_1^3 |x^2 - 4x + 3| dx = \\
 &= \int_1^3 -(x^2 - 4x + 3) dx = \frac{4}{3}
 \end{aligned}$$

4.



$$S = \int_0^2 [(-x^2 + 4x) - 2x] dx = \frac{4}{3}$$

5.



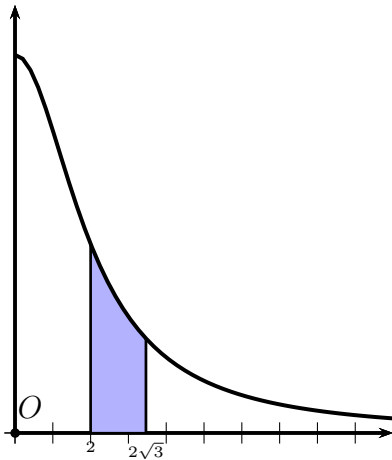
Respecto al eje  $X$ :

$$\begin{aligned} S &= \int_1^2 (x^2 - 1) dx + \int_2^8 \left( \frac{8}{x} - 1 \right) dx = \\ &= \frac{4}{3} + (16 \ln 2 - 6) = 16 \ln 2 - \frac{14}{3} \end{aligned}$$

Más fácil con respecto al eje  $Y$ :

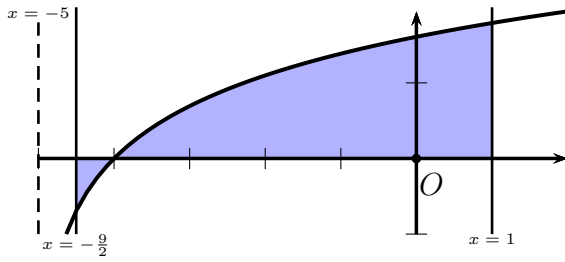
$$S = \int_1^4 \left( \frac{8}{y} - \sqrt{y} \right) dy = 16 \ln 2 - \frac{14}{3}$$

7.



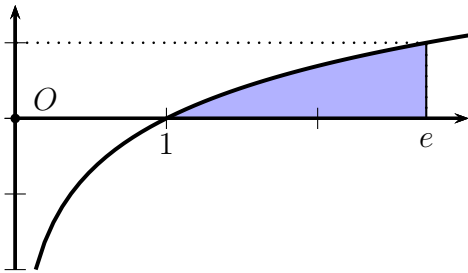
$$S = \int_2^{2\sqrt{3}} \frac{1}{4+x^2} dx = \frac{\pi}{24}$$

8.



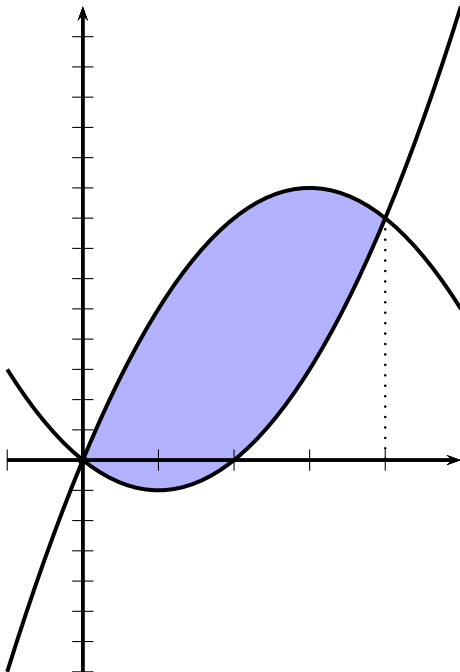
$$\begin{aligned}
 S &= \int_{-\frac{5}{2}}^{-4} -\ln(x+5) dx + \int_{-4}^1 \ln(x+5) dx = \\
 &= \left(-\frac{1}{2} \ln 2 + \frac{1}{2}\right) + (6 \ln 6 - 5) = \\
 &= 6 \ln 6 - \frac{1}{2} \ln 2 - \frac{9}{2}
 \end{aligned}$$

9.



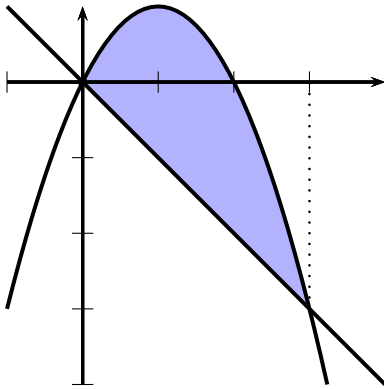
$$S = \int_1^e \ln x dx = 1$$

10.



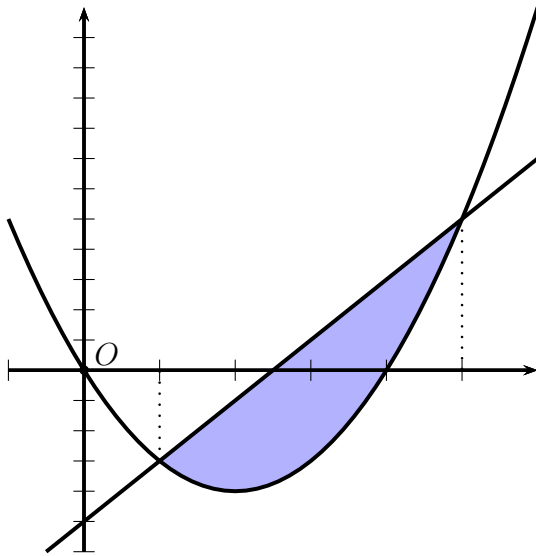
$$S = \int_0^4 [(6x - x^2) - (x^2 - 2x)] dx = \frac{64}{3}$$

12.



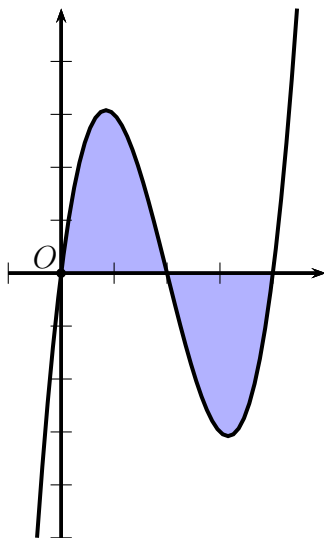
$$S = \int_0^3 [(2x - x^2) + x] dx = \frac{9}{2}$$

14.



$$S = \int_1^5 [(2x - 5) - (x^2 - 4x)] dx = \frac{32}{3}$$

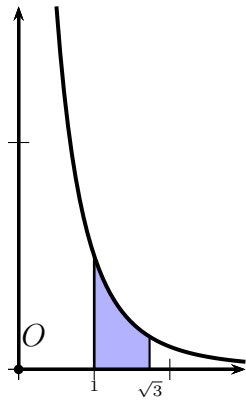
18.



$$\begin{aligned} S &= \int_0^2 (x^3 - 6x^2 + 8x) dx + \int_2^4 -(x^3 - 6x^2 + 8x) dx = \\ &= 4 + 4 = 8 \end{aligned}$$

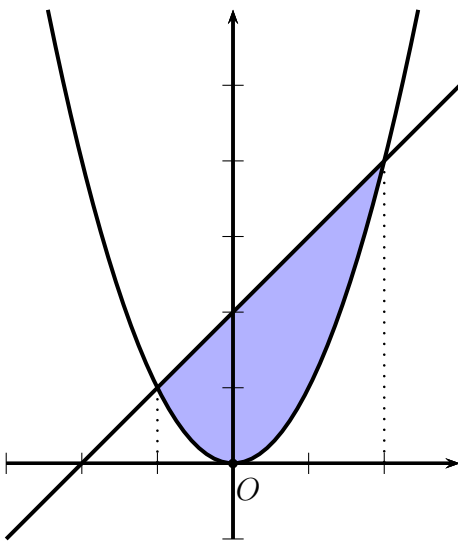


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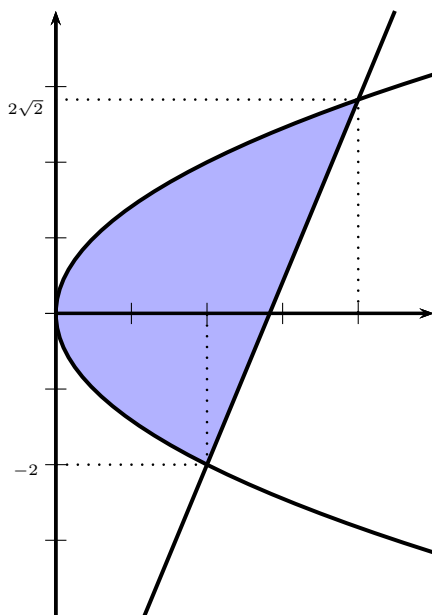
$$S = \int_1^{\sqrt{3}} \frac{1}{x(x^2 + 1)} dx = \frac{1}{2} \ln \left( \frac{3}{2} \right)$$

21.



$$S = \int_{-1}^2 (x + 2 - x^2) dx = \frac{9}{2}$$

22.



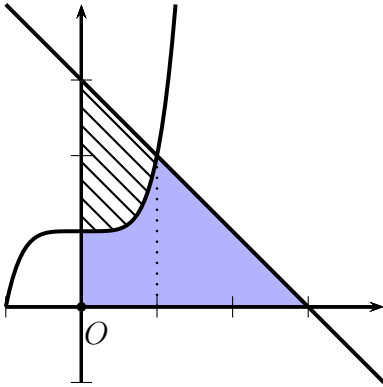
Recta  $\equiv y = -2 + (1 + \sqrt{2})(x - 2)$   
Respecto al eje X:

$$\begin{aligned} S &= \int_0^2 2\sqrt{2}x dx + \\ &+ \int_2^4 [\sqrt{2}x + 2 - (1 + \sqrt{2})(x - 2)] dx = \\ &= \left( \frac{16}{3} \right) + \left( \frac{10\sqrt{2} - 2}{3} \right) = \frac{10\sqrt{2} + 14}{3} \end{aligned}$$

Más fácil respecto al eje Y:

$$\begin{aligned} S &= \int_{-2}^{2\sqrt{2}} \left( 2 + \frac{y + 2}{1 + \sqrt{2}} - \frac{y^2}{2} \right) dy = \\ &= \frac{10\sqrt{2} + 14}{3} \end{aligned}$$

23.



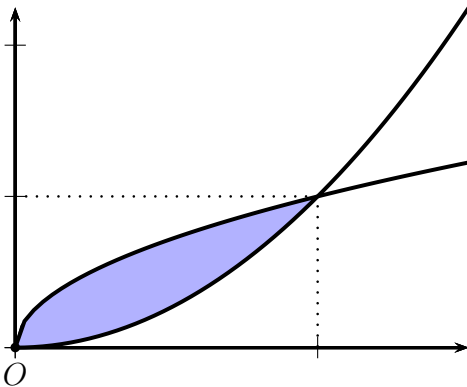
Sea  $S_1$  la superficie de la región rayada:

$$S_1 = \int_0^1 [(3-x) - (1+x^5)] dx = \frac{4}{3}$$

Sea  $S_2$  la superficie de la región sombreada:

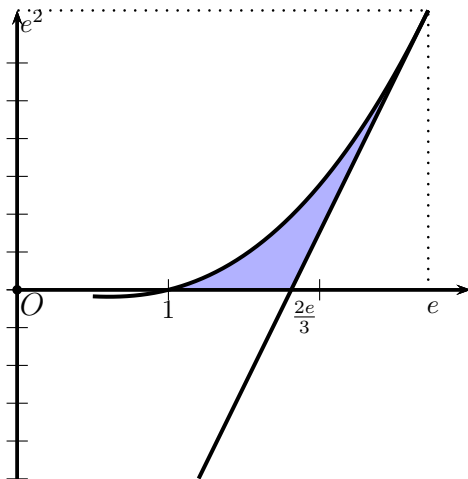
$$S_2 = \frac{9}{2} - \frac{4}{3} = \frac{19}{6}$$

24.



$$S = \int_0^1 (\sqrt{x} - x^2) dx = \frac{1}{3}$$

26.



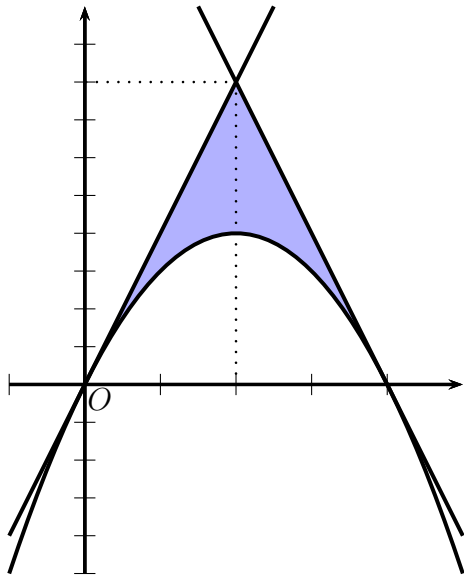
La recta tangente en  $x = e$  es:

$$y = 3ex - 2e^2$$

La superficie del triángulo de vértices  $(2e/3, 0)$ ,  $(e, 0)$ ,  $(e, e^2)$  es  $e^3/6$ , luego

$$S = \int_1^e x^2 \ln x dx - \frac{e^3}{6} = \frac{e^3 + 2}{18}$$

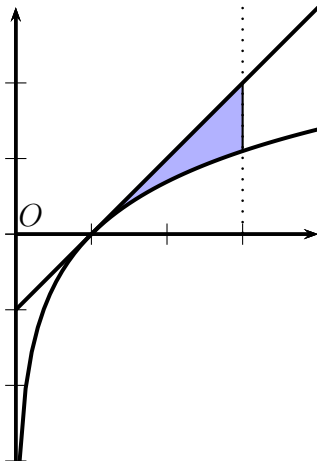
30.



La recta tangente en  $x = 0$  es  $y = 4x$ .  
La región es simétrica respecto de la recta  $x = 2$  ( demuéstrello), luego:

$$S = 2 \int_0^2 [4x - (4x - x^2)] dx = \frac{16}{3}$$

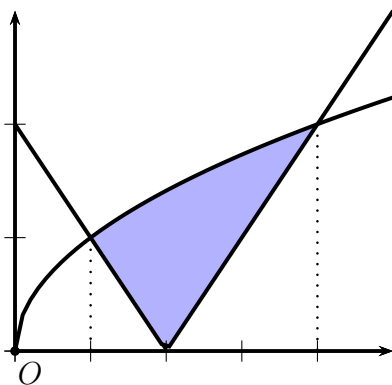
31.



La recta tangente en  $x = 1$  es  $y = x - 1$ .

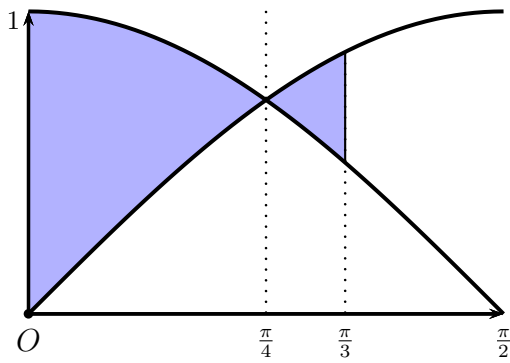
$$S = \int_1^3 [(x - 1) - \ln x] dx = 4 - 3 \ln 3$$

32.



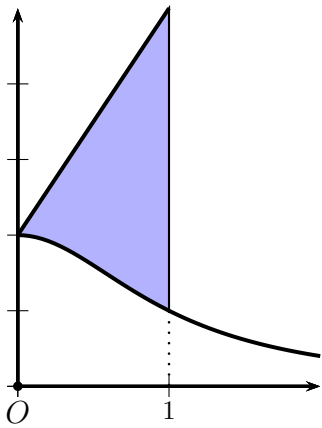
$$S = \int_1^2 [\sqrt{x} + (x - 2)] dx + \int_2^4 [\sqrt{x} - (x - 2)] dx = \frac{13}{6}$$

39.



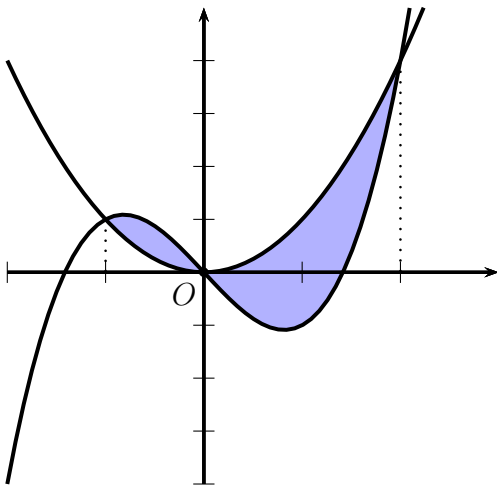
$$\begin{aligned}
 S &= \int_0^{\pi/4} (\cos x - \sin x) dx + \\
 &+ \int_{\pi/4}^{\pi/3} (\sin x - \cos x) dx = \\
 &= (\sqrt{2} - 1) + \left( \sqrt{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} \right) = \\
 &= 2\sqrt{2} - \frac{3}{2} - \frac{\sqrt{3}}{2}
 \end{aligned}$$

40.



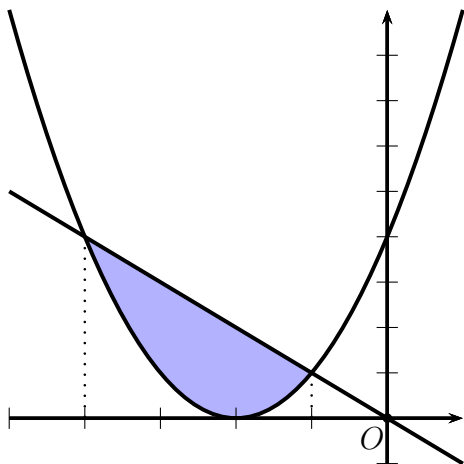
$$S = \int_0^1 \left[ (3x + 2) - \frac{2}{1 + x^2} \right] dx = \frac{7 - \pi}{2}$$

44.



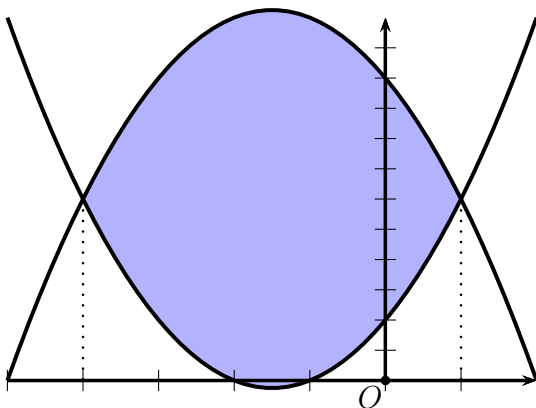
$$\begin{aligned}
 S &= \int_{-1}^0 [(x^3 - 2x) - x^2] dx + \\
 &+ \int_0^2 [x^2 - (x^3 - 2x)] dx = \\
 &= \frac{5}{12} + \frac{8}{3} = \frac{37}{12}
 \end{aligned}$$

46.



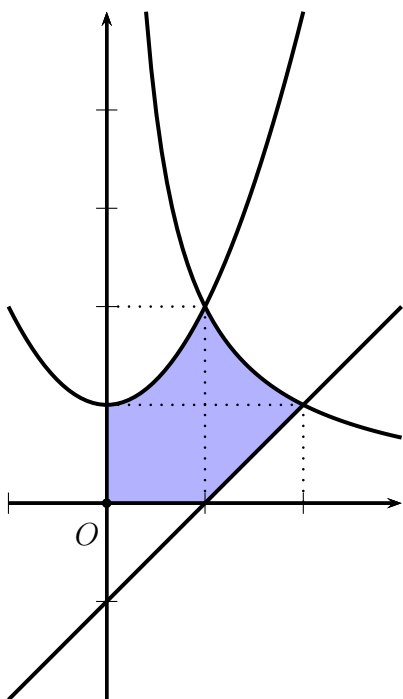
$$S = \int_{-4}^{-1} [-x - (x + 2)^2] dx = \frac{9}{2}$$

47.



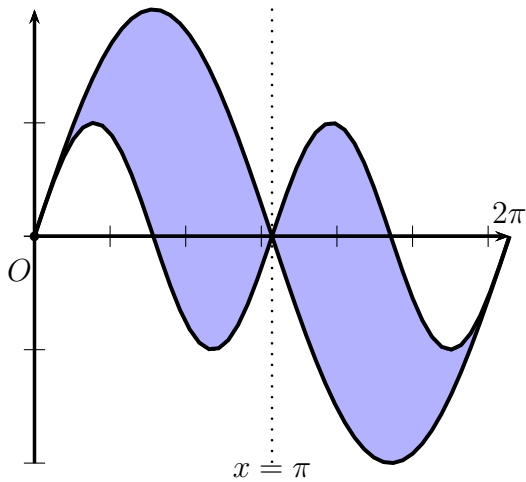
$$\begin{aligned} S &= \int_{-3}^3 [(-x^2 - 3x + 10) - (x^2 + 3x + 2)] dx = \\ &= \frac{125}{3} \end{aligned}$$

51.



$$\begin{aligned} S &= \int_0^1 (x^2 + 1) dx + \int_1^2 \left[ \frac{2}{x} - (x - 1) \right] dx = \\ &= \frac{4}{3} + \left( -\frac{1}{2} + 2 \ln 2 \right) = \frac{5}{6} + 2 \ln 2 \end{aligned}$$

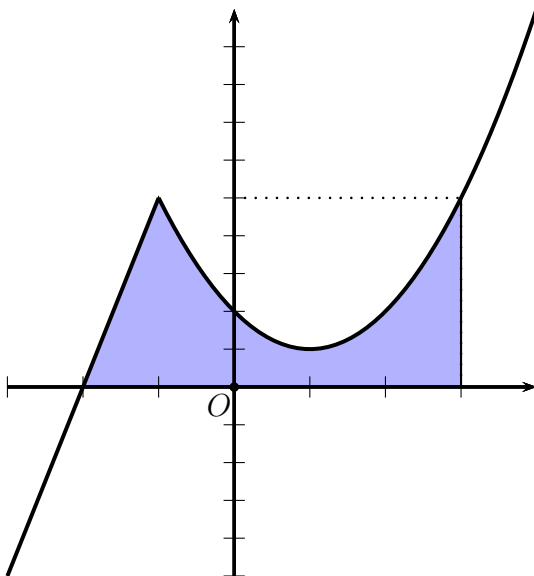
55.



Por simetria:

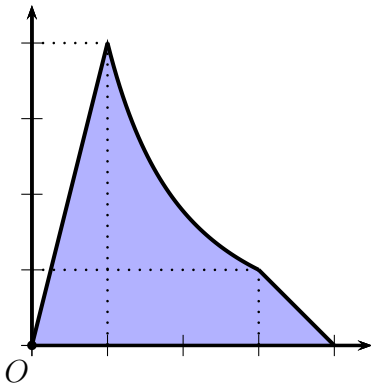
$$S = 2 \int_0^{\pi} (2 \operatorname{sen} x - \operatorname{sen} 2x) dx = 8$$

57.



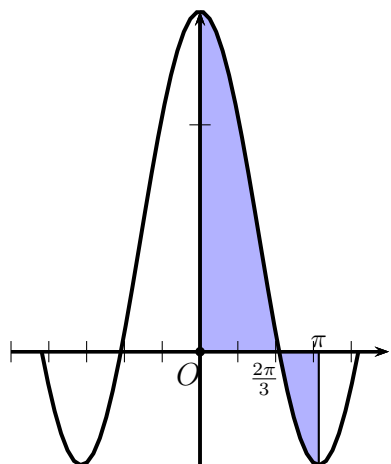
$$\begin{aligned} S &= \int_{-2}^{-1} (5x + 10) dx + \int_{-1}^3 (x^2 - 2x + 2) dx = \\ &= \frac{5}{2} + \frac{28}{3} = \frac{71}{6} \end{aligned}$$

62.



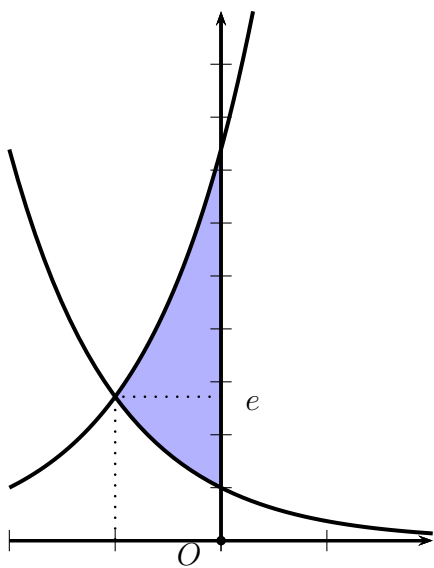
$$\begin{aligned} S &= \int_0^1 4x dx + \int_1^3 \frac{16}{(x+1)^2} dx + \int_3^4 (4-x) dx = \\ &= 2 + 4 + \frac{1}{2} = \frac{13}{2} \end{aligned}$$

63.



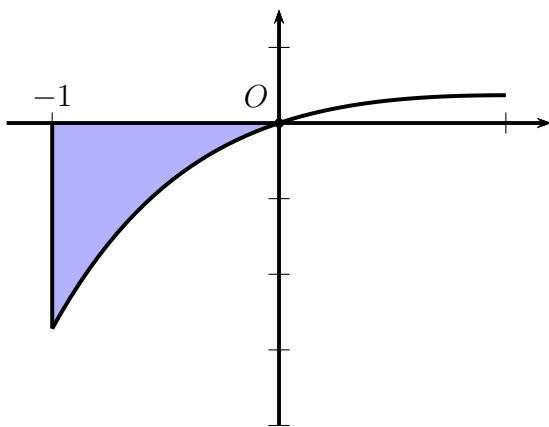
$$\begin{aligned}
 S &= \int_0^{2\pi/3} \left( \frac{1}{2} + \cos x \right) dx + \\
 &+ \int_{2\pi/3}^{\pi} - \left( \frac{1}{2} + \cos x \right) dx = \\
 &= \left( \frac{\pi}{3} + \frac{\sqrt{3}}{2} \right) + \left( -\frac{\pi}{6} + \frac{\sqrt{3}}{2} \right) = \\
 &= \sqrt{3} + \frac{\pi}{6}
 \end{aligned}$$

64.



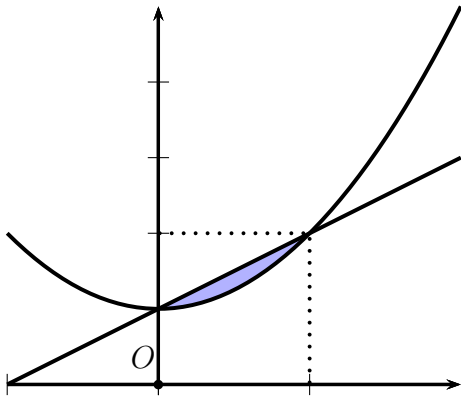
$$S = \int_{-1}^0 (e^{x+2} - e^{-x}) dx = (e - 1)^2$$

68.



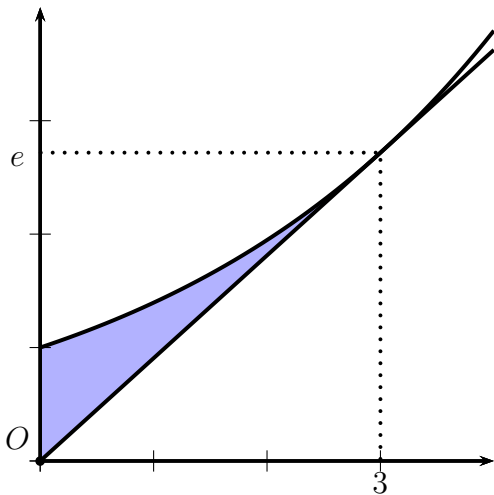
$$S = \int_{-1}^0 -xe^{-x} dx = 1$$

71.



$$S = \int_0^1 [(1+x) - (1+x^2)] dx = \frac{1}{6}$$

72.

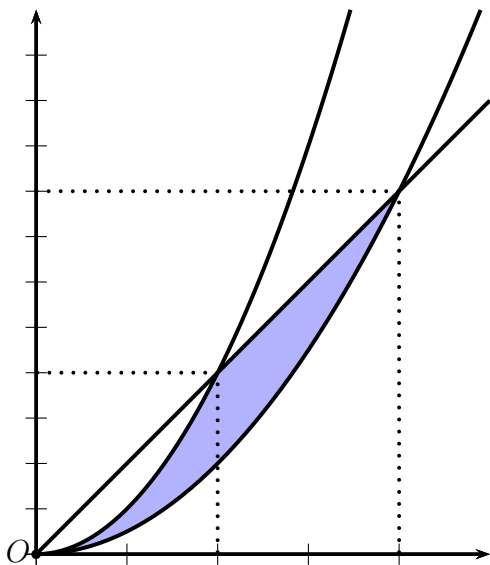


La recta tangente en  $x = 3$  es:

$$y = \frac{ex}{3}$$

$$S = \int_0^3 \left( e^{x/3} - \frac{ex}{3} \right) dx = \frac{3e}{2} - 3$$

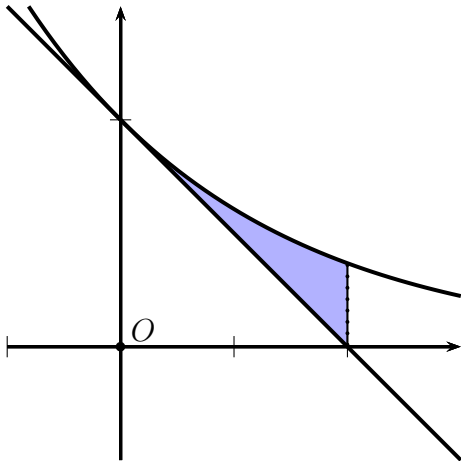
74.



$$\begin{aligned} S &= \int_0^2 \left( x^2 - \frac{x^2}{2} \right) dx + \int_2^4 \left( 2x - \frac{x^2}{2} \right) dx = \\ &= \frac{4}{3} + \frac{8}{3} = 4 \end{aligned}$$



77.

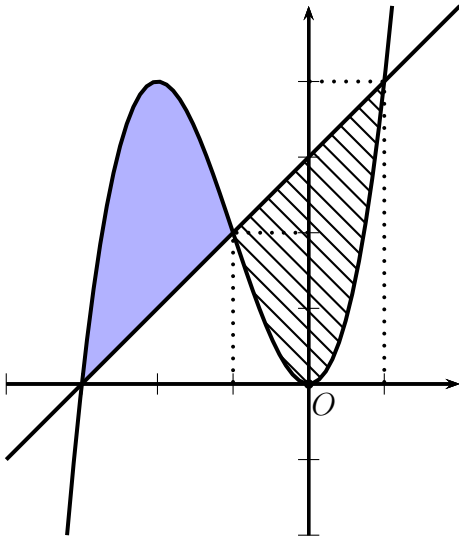


La recta tangente en  $x = 0$  es:

$$y = 1 - \frac{x}{2}$$

$$S = \int_0^2 \left[ e^{-x} - \left( 1 - \frac{x}{2} \right) \right] dx = 1 - \frac{2}{e}$$

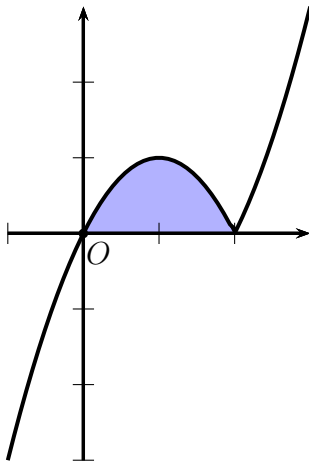
82.



$$S_{\text{sombreada}} = \int_{-3}^{-1} [(x^3 + 3x^2) - (x + 3)] dx = 4$$

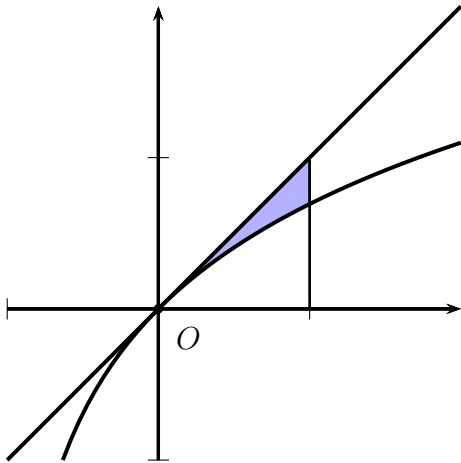
$$S_{\text{rayada}} = \int_{-1}^0 [(x + 3) - (x^3 + 3x^2)] dx = 4$$

83.



$$S = \int_0^2 x|x-2| dx = \int_0^2 -x(x-2) dx = \frac{4}{3}$$

84.

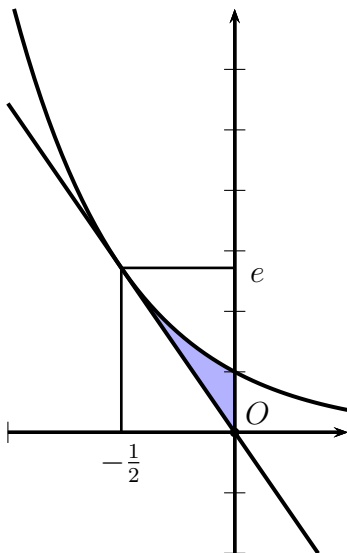


La recta tangente en  $x = 0$  es:

$$y = x$$

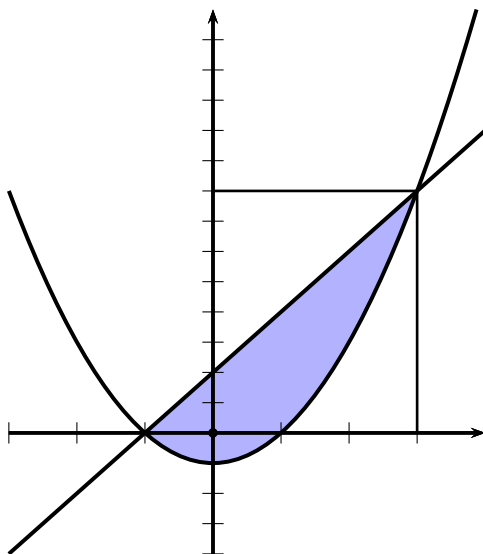
$$S = \int_0^1 [x - \ln(1+x)] dx = \frac{3}{2} - 2 \ln 2$$

86.



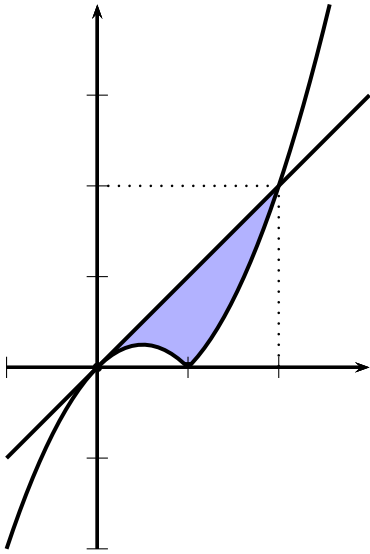
$$S = \int_{-\frac{1}{2}}^0 (e^{-2x} + 2ex) dx = \frac{e}{4} - \frac{1}{2}$$

88.



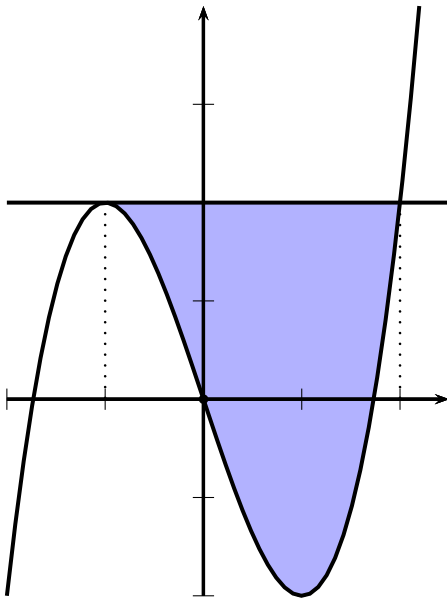
$$S = \int_{-1}^3 [(2x+2) - (x^2-1)] dx = \frac{32}{3}$$

89.



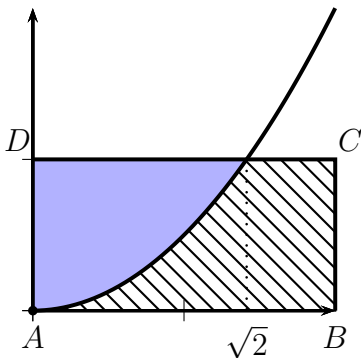
$$\begin{aligned}
 S &= \int_0^2 (x - x|x-1|) dx = \\
 &= \int_0^1 [x + x(x-1)] dx + \int_1^2 [x - x(x-1)] dx = \\
 &= \frac{1}{3} + \frac{2}{3} = 1
 \end{aligned}$$

90.



$$S = \int_{-1}^2 [2 - (x^3 - 3x)] dx = \frac{27}{4}$$

91.

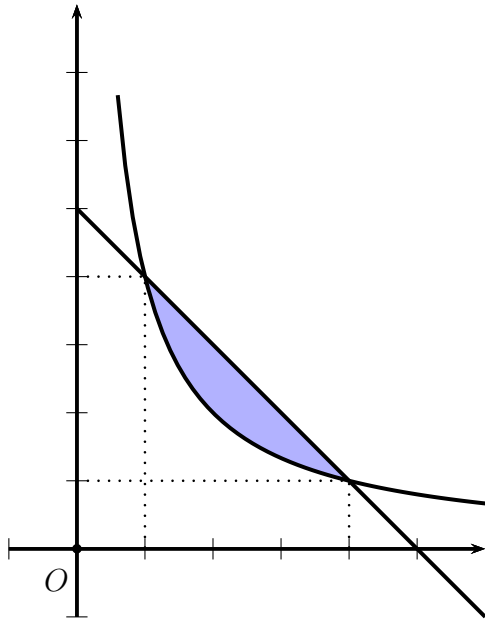


$$S_{\text{sombreada}} = \int_0^{\sqrt{2}} \left(1 - \frac{x^2}{2}\right) dx = \frac{2\sqrt{2}}{3}$$

La superficie del rectángulo  $ABCD$  es 2, luego

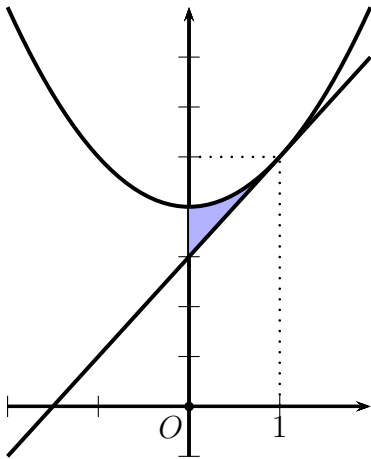
$$S_{\text{rayada}} = 2 - \frac{2\sqrt{2}}{3} = \frac{6 - 2\sqrt{2}}{3}$$

93.



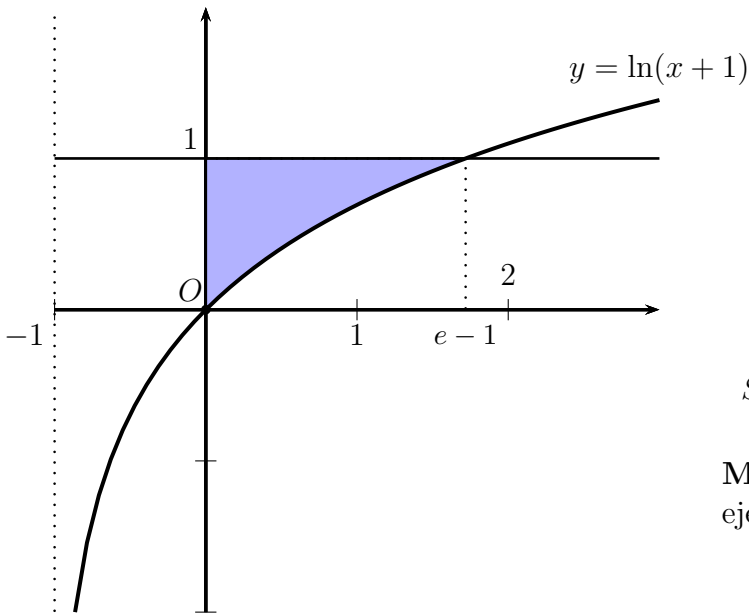
$$S = \int_1^4 \left( 5 - x - \frac{4}{x} \right) dx = \frac{15}{2} - 8 \ln 2$$

94.



$$S = \int_0^1 [x^2 + 4 - (2x + 3)] dx = \frac{1}{3}$$

95.

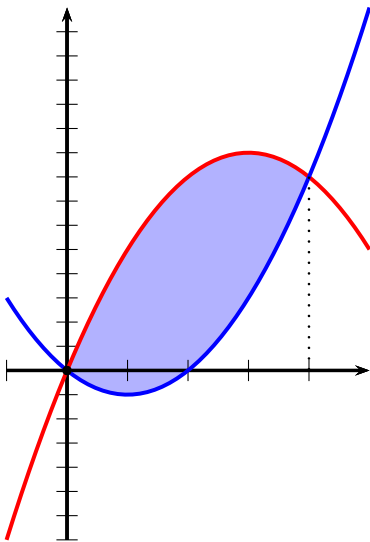


$$S = \int_0^{e-1} [1 - \ln(x+1)] dx = e - 2$$

**Muchísimo más fácil** con respecto al eje Y:

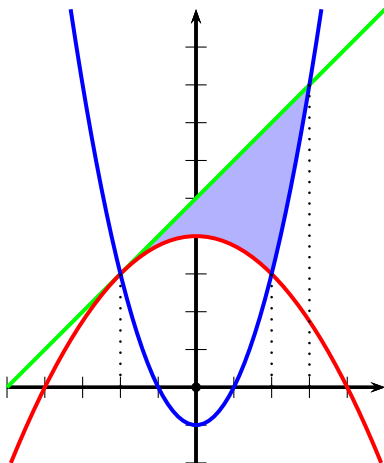
$$S = \int_0^1 (e^y - 1) dy = e - 2$$

96.



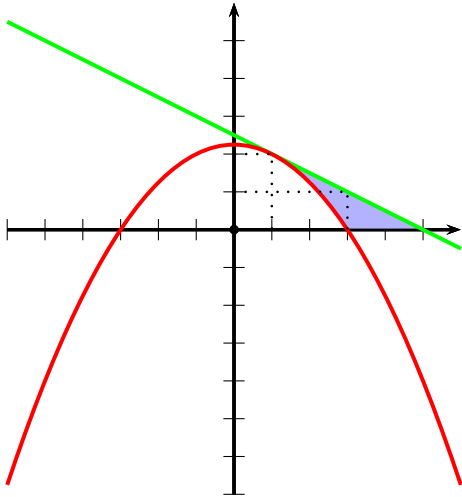
$$S = \int_0^4 [(6x - x^2) - (x^2 - 2x)] dx = \frac{64}{3}$$

97.



$$S = S_1 + S_2 = \int_{-2}^2 \left[ (x+5) - \left( 4 - \frac{x^2}{4} \right) \right] dx + \int_2^3 [(x+5) - (x^2 - 1)] dx = \frac{16}{3} + \frac{13}{6} = \frac{15}{2}$$

101.



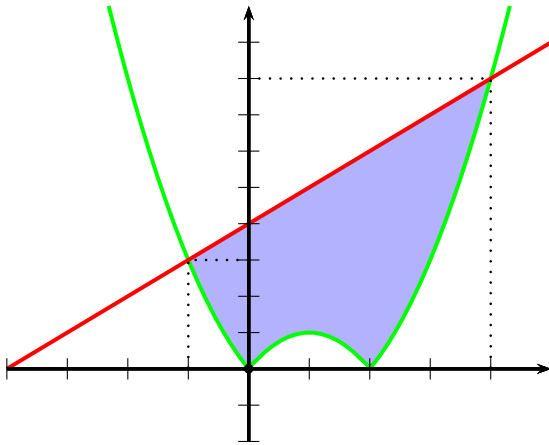
Integrando respecto al eje X:

$$S = S_1 + S_2 = \int_1^3 \left( \frac{5-x}{2} - \frac{9-x^2}{4} \right) dx + \int_3^5 \frac{5-x}{2} dx = \frac{2}{3} + 1 = \frac{5}{3}$$

Integrando respecto al eje Y:

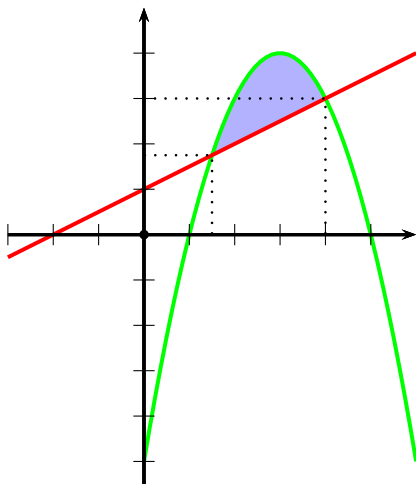
$$S = \int_0^2 \left[ (5-2y) - \sqrt{9-4y} \right] dy = \frac{5}{3}$$

102.



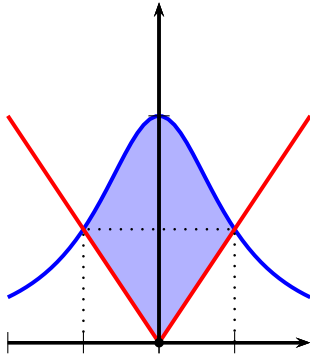
$$\begin{aligned} S &= \int_{-1}^4 [x+4 - |x \cdot (x-2)|] dx = \\ &= \int_{-1}^0 [x+4 - x(x-2)] dx + \\ &+ \int_0^2 [x+4 + x(x-2)] dx + \\ &+ \int_2^4 [x+4 - x(x-2)] dx = \\ &= \frac{13}{6} + \frac{26}{3} + \frac{22}{3} = \frac{109}{6} \end{aligned}$$

103.



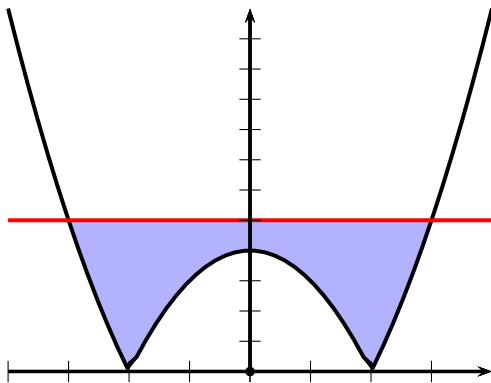
$$\begin{aligned} S &= \int_{3/2}^4 \left( -x^2 + 6x - 5 - \frac{x+2}{2} \right) dx = \\ &= \frac{125}{48} \end{aligned}$$

104.



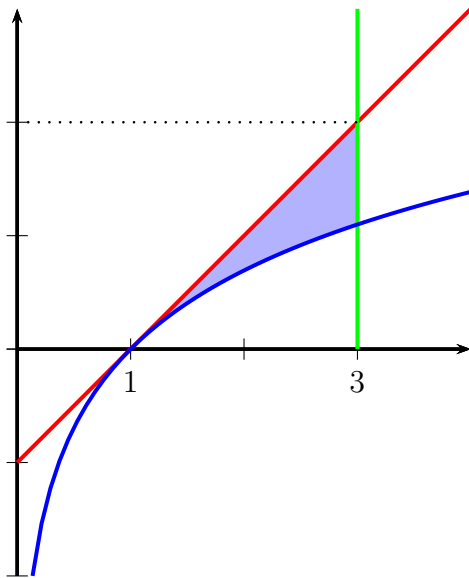
$$S = 2 \int_0^1 \left( \frac{1}{1+x^2} - \frac{x}{2} \right) dx = \frac{\pi - 1}{2}$$

109.



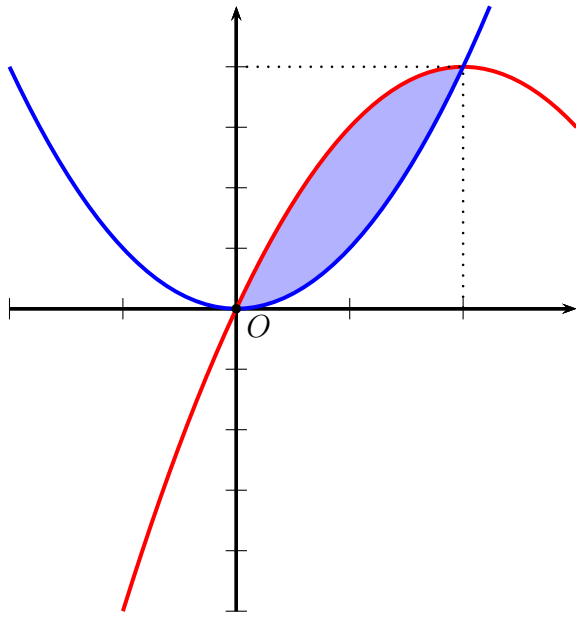
Sea  $f(x) = |x^2 - 4|$ . Entonces:  $S = 2(I_1 + I_2)$ ,  $I_1 = \int_0^2 (5 - f(x)) dx = \frac{14}{3}$ ,  $I_2 = \int_2^3 (5 - f(x)) dx = \frac{8}{3}$ , y de aquí  $S = \frac{44}{3}$ .

110.



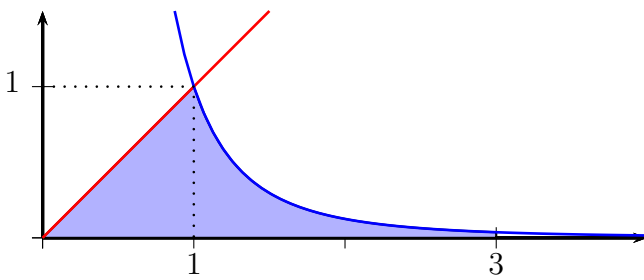
$$S = \int_1^3 (x - 1 - \ln x) dx = 4 - 3 \ln 3$$

112.



$$S = \int_0^2 [(-x^2 + 4x) - x^2] dx = \frac{8}{3}$$

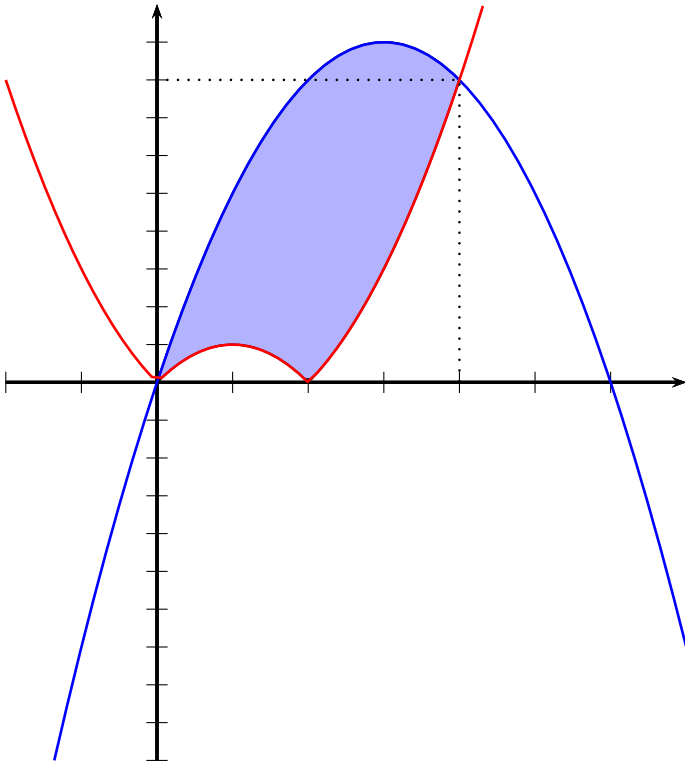
114.



$$S = \int_0^1 x dx + \int_1^3 \frac{1}{x^3} dx = \frac{17}{18}$$

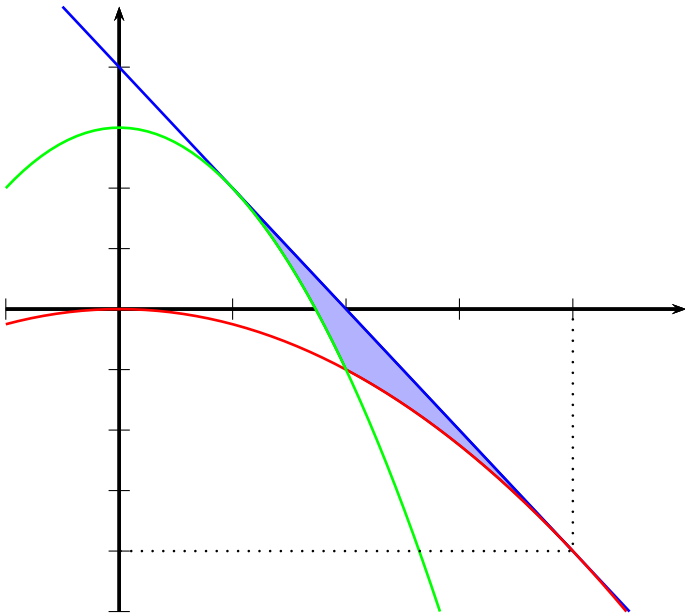
115.





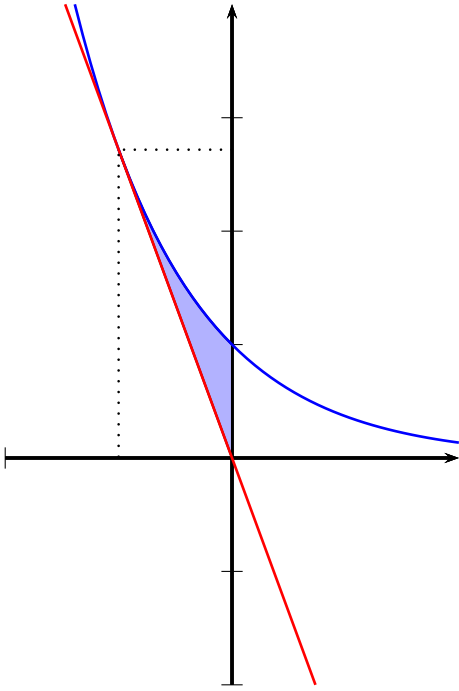
$$\begin{aligned}
 S &= \int_0^4 [(6x - x^2) - |x^2 - 2x|] dx = \\
 &= \int_0^2 [(6x - x^2) - (2x - x^2)] dx + \\
 &\int_2^4 [(6x - x^2) - (x^2 - 2x)] dx = \\
 &= 8 + \frac{32}{3} = \frac{56}{3}
 \end{aligned}$$

116.



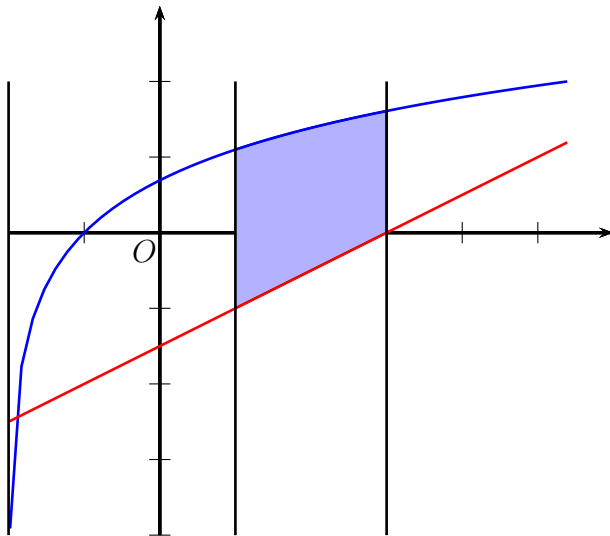
$$\begin{aligned}
 S &= \int_1^2 [(4 - 2x) - (3 - x^2)] dx + \\
 &\int_2^4 \left[ (4 - 2x) - \left( -\frac{x^2}{4} \right) \right] dx = \\
 &= \frac{1}{3} + \frac{2}{3} = 1
 \end{aligned}$$

118.



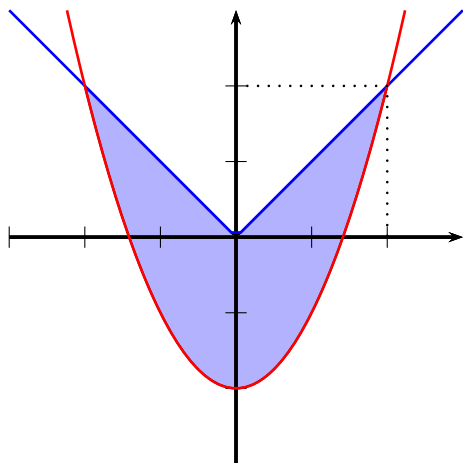
$$\begin{aligned}
 S &= \int_{-\frac{1}{2}}^0 (e^{-2x} + 2ex) dx = \\
 &= \left[ -\frac{1}{2}e^{-2x} + ex^2 \right]_{-\frac{1}{2}}^0 = \frac{e-2}{4}
 \end{aligned}$$

119.



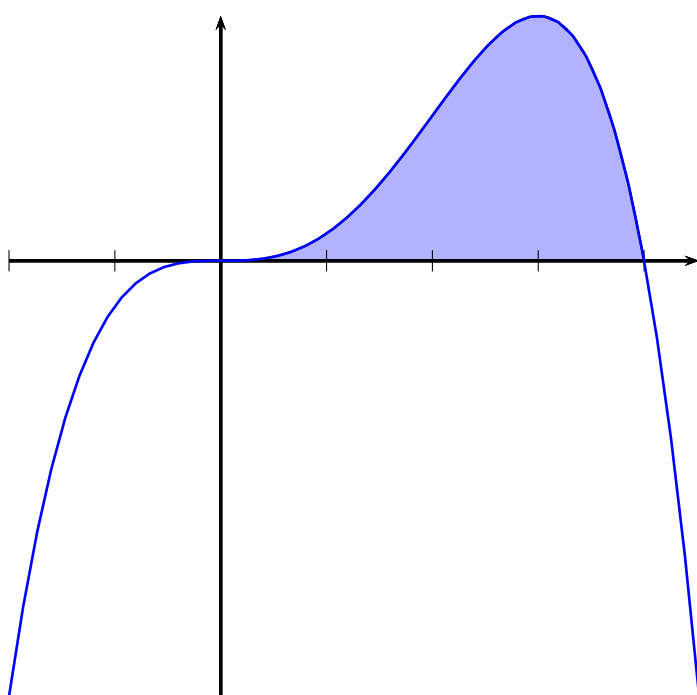
$$\begin{aligned}
 S &= \int_1^3 \left[ \ln(x+2) - \frac{x-3}{2} \right] dx = \\
 &= 5 \ln 5 - 3 \ln 3 - 1
 \end{aligned}$$

123.



$$S = 2 \int_0^2 [x - (x^2 - 2)] dx = \frac{20}{3}$$

124.



$$S = \int_0^4 (4x^3 - x^4) dx = \frac{256}{5}$$